# 314, March 20 '07-QUIZ 3

Name:______________________________________________

March 20, 2007

## 1 $< 13pts >$

Given the vectors

$$\mathbf{x}_1 = \begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix} , \mathbf{x}_2 = \begin{bmatrix} -1 \\ 2 \\ 4 \end{bmatrix} , \mathbf{x}_3 = \begin{bmatrix} -1 \\ 2 \\ 12 \end{bmatrix}$$

Give the dimension of Span$(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3)$ and find a basis.

**Solution**
Form the matrix

$$\begin{matrix} \\ 2 \\ -4 \end{matrix} \begin{pmatrix} 1 & -1 & -1 \\ -2 & 2 & 2 \\ 4 & 4 & 12 \end{pmatrix} \quad \rightarrow \quad \begin{matrix} \\ 2 \\ -4 \end{matrix} \begin{pmatrix} 1 & -1 & -1 \\ 0 & 0 & 0 \\ 0 & 8 & 16 \end{pmatrix}$$

There are pivots on columns 1 and 2, therefore the vectors $\mathbf{x}_1$ and $\mathbf{x}_2$ are linearly independent, providing a basis for Span$(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3)$, which is thus 2-dimensional.

## 2 $< 12pts >$

Given

$$\mathbf{v}_1 = \begin{bmatrix} 2 \\ 3 \end{bmatrix} , \mathbf{v}_2 = \begin{bmatrix} -1 \\ 2 \end{bmatrix} , S = \begin{bmatrix} 3 & -1 \\ 1 & 3 \end{bmatrix} .$$

Find vectors $\mathbf{u}_1$ , $\mathbf{u}_2$ so that S is the transition matrix from $[\mathbf{u}_1 , \mathbf{u}_2]$ to $[\mathbf{v}_1 , \mathbf{v}_2]$.

**Solution**

$$S = V^{-1}U \Rightarrow U = VS = \begin{pmatrix} 2 & -1 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} 3 & -1 \\ 1 & 3 \end{pmatrix} = \begin{pmatrix} 5 & -5 \\ 11 & 3 \end{pmatrix}$$

so

$$\mathbf{u}_1 = \begin{bmatrix} 5 \\ 11 \end{bmatrix} , \ \mathbf{u}_2 = \begin{bmatrix} -5 \\ 3 \end{bmatrix}$$

## 3 < 13*pts* >

Find a basis of the row space, column space and null space of

$$A = \begin{bmatrix} 1 & -1 & 0 & 1 & 2 \\ 2 & -1 & 1 & 3 & 1 \\ -1 & 2 & 1 & -1 & -3 \end{bmatrix}$$

and give the dimension of each.

**Solution** (use reduced row echelon form to find null vectors most easily)

$$\begin{matrix} \\ -2 \\ 1 \end{matrix} \begin{bmatrix} 1 & -1 & 0 & 1 & 2 \\ 2 & -1 & 1 & 3 & 1 \\ -1 & 2 & 1 & -1 & -3 \end{bmatrix} \rightarrow \begin{matrix} \\ \\ -1 \end{matrix} \begin{bmatrix} 1 & -1 & 0 & 1 & 2 \\ 0 & 1 & 1 & 1 & -3 \\ 0 & 1 & 1 & 0 & -1 \end{bmatrix} \rightarrow$$

$$\begin{matrix} \\ \\ -1* \end{matrix} \begin{bmatrix} 1 & -1 & 0 & 1 & 2 \\ 0 & 1 & 1 & 1 & -3 \\ 0 & 0 & 0 & -1 & 2 \end{bmatrix} \rightarrow \begin{matrix} -1 \\ -1 \\ \\ \end{matrix} \begin{bmatrix} 1 & -1 & 0 & 1 & 2 \\ 0 & 1 & 1 & 1 & -3 \\ 0 & 0 & 0 & 1 & -2 \end{bmatrix} \rightarrow$$

$$\begin{matrix} 1 \\ \\ \\ \end{matrix} \begin{bmatrix} 1 & -1 & 0 & 0 & 4 \\ 0 & 1 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & -2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 1 & 0 & 3 \\ 0 & 1 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & -2 \end{bmatrix}$$

so the pivots are found on columns 1, 2, 4 and so $x_3$ , $x_5$ are free variables. Therefore the dimensions of the column and row spaces are equal to 3, with the rows of $A$ giving the basis of the row space, $\mathcal{R}(A^T)$, and columns 1,2, and 4 of $A$ giving the basis of the column space, $\mathcal{R}(A)$. The dimension of the null space is 2. To find the basis of the null space:

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 3 \\ 0 & 1 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = 0 \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_4 \end{bmatrix} = -x_3 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} - x_5 \begin{bmatrix} 3 \\ -1 \\ -2 \end{bmatrix}$$

so that the null vectors (basis of $\mathcal{N}(A)$) are:

$$(x_3 = 1 \ , \ x_5 = 0 :) \ \mathbf{n}_1 = \begin{bmatrix} -1 \\ -1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \ , \ (x_3 = 0 \ , \ x_5 = 1 :) \ \mathbf{n}_2 = \begin{bmatrix} -3 \\ 1 \\ 0 \\ 2 \\ 1 \end{bmatrix}$$