314 '03-QUIZ 2

Name:____

October 9, 2003

1 $< 12pts >$

Given

$$A = \begin{pmatrix} 2 & 1 & 1 \\ 4 & 1 & 3 \\ 6 & 4 & 5 \end{pmatrix}$$

find elementary matrices E_1, E_2, such that $E_2 E_1 A = U$ where U is upper triangular.

Solution

Carry out elimination on augmented matrix of part 2, to get both problems in one fell swoop:

$$\begin{matrix} \\ -2 \\ -3 \end{matrix} \left(\begin{array}{ccc|ccc} 2 & 1 & 1 & 1 & 0 & 0 \\ 4 & 1 & 3 & 0 & 1 & 0 \\ 6 & 4 & 5 & 0 & 0 & 1 \end{array} \right) \to \begin{matrix} \\ \\ 1 \end{matrix} \left(\begin{array}{ccc|ccc} 2 & 1 & 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & -2 & 1 & 0 \\ 0 & 1 & 2 & -3 & 0 & 1 \end{array} \right) \to$$

$$\left(\begin{array}{ccc|ccc} 2 & 1 & 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & -2 & 1 & 0 \\ 0 & 0 & 3 & -5 & 1 & 1 \end{array} \right)$$

so that we can easily read out the elementary matrices and U matrix:

$$E_2 E_1 A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -3 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & 1 & 1 \\ 4 & 1 & 3 \\ 6 & 4 & 5 \end{pmatrix} = \begin{pmatrix} 2 & 1 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 3 \end{pmatrix} = U\ .$$

2 $< 13pts >$

For the matrix in the previous problem, solve the system

$$Ax = e_i, \ , \ i = 1, 2, 3$$

with

$$e_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \ , \ e_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \ , \ e_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \ .$$

(That is find the inverse of A! You may solve all three problems by using a combined augmented matrix $B = (A|I)$ where I is the identity).

Solution

We continue with the Gauss-Jordan elimination:

$$\begin{array}{r} 1/2* \\ -1* \\ 1/3* \end{array} \left(\begin{array}{rrr|rrr} 2 & 1 & 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & -2 & 1 & 0 \\ 0 & 0 & 3 & -5 & 1 & 1 \end{array} \right) \rightarrow$$

$$\begin{array}{r} -1/2 \\ 1 \\ \ \end{array} \left(\begin{array}{rrr|rrr} 1 & 1/2 & 1/2 & 1/2 & 0 & 0 \\ 0 & 1 & -1 & 2 & -1 & 0 \\ 0 & 0 & 1 & -5/3 & 1/3 & 1/3 \end{array} \right) \rightarrow$$

$$\begin{array}{r} -1/2 \\ \ \\ \ \end{array} \left(\begin{array}{rrr|rrr} 1 & 1/2 & 0 & 4/3 & -1/6 & -1/6 \\ 0 & 1 & 0 & 1/3 & -2/3 & 1/3 \\ 0 & 0 & 1 & -5/3 & 1/3 & 1/3 \end{array} \right) \rightarrow \left(\begin{array}{rrr|rrr} 1 & 0 & 0 & 7/6 & 1/6 & -1/3 \\ 0 & 1 & 0 & 1/3 & -2/3 & 1/3 \\ 0 & 0 & 1 & -5/3 & 1/3 & 1/3 \end{array} \right) .$$

so that

$$A^{-1} = \frac{1}{6} \begin{pmatrix} 7 & 1 & -2 \\ 2 & -4 & 2 \\ -10 & 2 & 2 \end{pmatrix} .$$