

314 '03-QUIZ 3

Name:_____

October 12, 2003

1 < 12pts >

Find the inverse of the matrix

$$A = \begin{pmatrix} -1 & -3 & -3 \\ 2 & 6 & 1 \\ 3 & 8 & 3 \end{pmatrix}$$

Solution

$$\begin{array}{l} 2 \\ 3 \end{array} \left(\begin{array}{ccc|ccc} -1 & -3 & -3 & 1 & 0 & 0 \\ 2 & 6 & 1 & 0 & 1 & 0 \\ 3 & 8 & 3 & 0 & 0 & 1 \end{array} \right) \rightarrow \left(\begin{array}{ccc|ccc} -1 & -3 & -3 & 1 & 0 & 0 \\ 0 & 0 & -5 & 2 & 1 & 0 \\ 0 & -1 & -6 & 3 & 0 & 1 \end{array} \right)$$

Now permute rows 2 and 3:

$$\begin{array}{l} -1* \\ -1* \\ -1/5* \end{array} \left(\begin{array}{ccc|ccc} -1 & -3 & -3 & 1 & 0 & 0 \\ 0 & -1 & -6 & 3 & 0 & 1 \\ 0 & 0 & -5 & 2 & 1 & 0 \end{array} \right) \rightarrow \begin{array}{l} -3 \\ -6 \\ \end{array} \left(\begin{array}{ccc|ccc} 1 & 3 & 3 & -1 & 0 & 0 \\ 0 & 1 & 6 & -3 & 0 & -1 \\ 0 & 0 & 1 & -2/5 & -1/5 & 0 \end{array} \right)$$

$$\rightarrow \begin{array}{l} -3 \\ \\ \end{array} \left(\begin{array}{ccc|ccc} 1 & 3 & 0 & 1/5 & 3/5 & 0 \\ 0 & 1 & 0 & -3/5 & 6/5 & -1 \\ 0 & 0 & 1 & -2/5 & -1/5 & 0 \end{array} \right) \rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & -3 & 3 \\ 0 & 1 & 0 & -3/5 & 6/5 & -1 \\ 0 & 0 & 1 & -2/5 & -1/5 & 0 \end{array} \right)$$

$$A^{-1} = \begin{pmatrix} 2 & -3 & 3 \\ -3/5 & 6/5 & -1 \\ -2/5 & -1/5 & 0 \end{pmatrix}$$

2 < 13pts >

Find the LU factorization and evaluate the determinant of the matrix

$$A = \begin{pmatrix} -2 & 1 & 2 \\ 4 & 1 & -2 \\ -6 & -3 & 4 \end{pmatrix}$$

Solution

$$\begin{matrix} 2 \\ -3 \end{matrix} \begin{pmatrix} -2 & 1 & 2 \\ 4 & 1 & -2 \\ -6 & -3 & 4 \end{pmatrix} \rightarrow \begin{matrix} 2 \\ 2 \end{matrix} \begin{pmatrix} -2 & 1 & 2 \\ 0 & 3 & 2 \\ 0 & 0 & 2 \end{pmatrix} = U$$

Then

$$L = E_1^{-1}E_2^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 3 & -2 & 1 \end{pmatrix}$$

The determinant is simply the product of the diagonal elements of U , that is

$$\det A = \det U = (-2) * (3) * (2) = -12 .$$