314 '03-QUIZ 5

Name:____

October 9, 2003

1 $< 12pts >$

Let $\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n\}$ be a spanning set for the vector space V, and let $\mathbf{v}$ be any other vector in V. Show that $\mathbf{v}, \mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n$ are linearly dependent.

Solution

- Since the set $S = \{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n\}$ spans V and $\mathbf{v} \in V$ while $\mathbf{v} \notin S$ we have, byt the definition of a spanning set, that there exist numbers C_i , $i = 1, \ldots, n$ such that

$$\mathbf{v} = C_1\mathbf{v}_1 + \cdots + C_n\mathbf{v}_n \ .$$

- Now rewrite above relationship as:

$$(-1) * \mathbf{v} + C_1\mathbf{v}_1 + \cdots + C_n\mathbf{v}_n = \mathbf{0} \ .$$

  In this relationship we have a combination of the vectors $\mathbf{v}, \mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n$ that is equal to zero, while at least one coefficient (here the coefficient (-1) multiplying the vector $\mathbf{v}$) is not equal to zero. By the definition of a linearly dependent set, this set is dependent.

2 $< 13pts >$

Given the vectors

$$\mathbf{x}_1 = \begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix}, \; \mathbf{x}_2 = \begin{bmatrix} -1 \\ 2 \\ 4 \end{bmatrix}, \; \mathbf{x}_3 = \begin{bmatrix} -1 \\ 2 \\ 12 \end{bmatrix}$$

Give the dimension of $\text{Span}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3)$ and find a basis.

Solution
Form the matrix

$$\begin{matrix} \\ 2 \\ -4 \end{matrix} \begin{pmatrix} 1 & -1 & -1 \\ -2 & 2 & 2 \\ 4 & 4 & 12 \end{pmatrix} \quad \rightarrow \quad \begin{matrix} \\ 2 \\ -4 \end{matrix} \begin{pmatrix} 1 & -1 & -1 \\ 0 & 0 & 0 \\ 0 & 8 & 16 \end{pmatrix}$$

There are pivots on columns 1 and 2, therefore the vectors $\mathbf{x}_1$ and $\mathbf{x}_2$ are linearly independent, providing a basis for $\text{Span}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3)$, which is thus 2-dimensional.