314 '03-QUIZ 8

Name:____

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1 $< 12pts >$

Find all least squares solutions of the system

$$\begin{bmatrix} 1 & 3 & 1 \\ 2 & 4 & 1 \\ 3 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2 \\ 8 \\ 0 \end{bmatrix}$$

Solution
First form the normal equations, $A^T A x = A^T b$:

$$\begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 1 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 3 & 1 \\ 2 & 4 & 1 \\ 3 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 1 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} -2 \\ 8 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 14 & 14 & 0 \\ 14 & 26 & 6 \\ 0 & 6 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 14 \\ 26 \\ 6 \end{bmatrix}$$

Form augmented matrix and proceed as usual when there are free variables:

$$\begin{array}{c} \\ -1 \\ \\ \end{array} \left[\begin{array}{ccc|c} 14 & 14 & 0 & 14 \\ 14 & 26 & 6 & 26 \\ 0 & 6 & 3 & 6 \end{array}\right] \rightarrow \begin{array}{c} \\ \\ -1/2 \end{array} \left[\begin{array}{ccc|c} 14 & 14 & 0 & 14 \\ 0 & 12 & 6 & 12 \\ 0 & 6 & 3 & 6 \end{array}\right] \rightarrow \begin{array}{c} 1/14 \\ 1/12 \\ \\ \end{array} \left[\begin{array}{ccc|c} 14 & 14 & 0 & 14 \\ 0 & 12 & 6 & 12 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & 1 & 1/2 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right] \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + \alpha \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$$

2 $< 13pts >$

Find the least squares fit by a linear function ($y = a * x + b$) to the data

x	-1	0	1	2
y	0	2	5	16

Solution

We have $a * x_i + b = y_i$, $i = 1, 2, 3, 4$. Writing as a system:

$$\begin{bmatrix} -1 & 1 \\ 0 & 1 \\ 1 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 5 \\ 16 \end{bmatrix}$$

Multiplying by the transpose of the matrix:

$$\begin{bmatrix} -1 & 0 & 1 & 2 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ 0 & 1 \\ 1 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 37 \\ 23 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 & 2 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ 5 \\ 16 \end{bmatrix}$$

Since

$$\begin{bmatrix} 6 & 2 \\ 2 & 4 \end{bmatrix}^{-1} = \frac{1}{6 * 4 - 2 * 2} \begin{bmatrix} 4 & -2 \\ -2 & 6 \end{bmatrix} = \frac{1}{20} \begin{bmatrix} 4 & -2 \\ -2 & 6 \end{bmatrix}$$

we find

$$\begin{bmatrix} a \\ b \end{bmatrix} = \frac{1}{20} \begin{bmatrix} 4 & -2 \\ -2 & 6 \end{bmatrix} \begin{bmatrix} 37 \\ 23 \end{bmatrix} = \begin{bmatrix} 102/20 \\ 64/20 \end{bmatrix} = \begin{bmatrix} 5.1 \\ 3.2 \end{bmatrix}$$