

Math. 314
Fall '99
Set I

$$(6e) \begin{cases} 2x_1 + x_2 + 3x_3 = 1 \\ 4x_1 + 3x_2 + 5x_3 = 1 \\ 6x_1 + 5x_2 + 5x_3 = -3 \end{cases}$$

p.11 6(e,h), 8(a,b)

$$\begin{pmatrix} 2 & 1 & 3 & | & 1 \\ 4 & 3 & 5 & | & 1 \\ -3 & 6 & 5 & | & -3 \end{pmatrix} \xrightarrow{-2} \begin{pmatrix} 2 & 1 & 3 & | & 1 \\ 0 & 1 & -1 & | & -1 \\ 0 & 2 & -4 & | & -6 \end{pmatrix} \rightarrow \begin{pmatrix} 2 & 1 & 3 & | & 1 \\ 0 & 1 & -1 & | & -1 \\ 0 & 0 & -2 & | & -4 \end{pmatrix}$$

Triangular system:

$$\begin{aligned} 2x_1 + x_2 + 3x_3 &= 1 \Rightarrow x_1 = \frac{1}{2}(1 - x_2 - 3x_3) = \frac{1}{2}(-6) \\ x_2 - x_3 &= -1 \Rightarrow x_2 = -1 + x_3 = 1 \\ -2x_3 &= -4 \Rightarrow x_3 = 2 \end{aligned}$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -3 \\ 1 \\ 2 \end{pmatrix}$$

$$(6h) \begin{pmatrix} 0 & 1 & 1 & 1 & | & 0 \\ 3 & 0 & 3 & -4 & | & 7 \\ 1 & 1 & 1 & 2 & | & 6 \\ 2 & 3 & 1 & 3 & | & 6 \end{pmatrix} \xrightarrow{-3} \begin{pmatrix} 0 & 1 & 1 & 1 & | & 0 \\ 3 & 0 & 3 & -4 & | & 7 \\ 0 & 1 & 1 & 1 & | & 6 \\ -2 & 2 & 3 & 1 & | & 6 \end{pmatrix} \rightarrow$$

$$\rightarrow \begin{pmatrix} 1 & 1 & 1 & 2 & | & 6 \\ 0 & -3 & 0 & -10 & | & -11 \\ 0 & 1 & 1 & 1 & | & 0 \\ 0 & 1 & -1 & -1 & | & -6 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & 1 & 2 & | & 6 \\ 0 & 1 & 1 & 1 & | & 0 \\ 0 & -3 & 0 & -10 & | & -11 \\ -1 & 0 & 1 & -1 & | & -6 \end{pmatrix} \rightarrow$$

$$\begin{pmatrix} 1 & 1 & 1 & 2 & | & 6 \\ 0 & 1 & 1 & 1 & | & 0 \\ 0 & 0 & 3 & -7 & | & -11 \\ 0 & 0 & -2 & -2 & | & -6 \end{pmatrix}$$

scale last row by $-1/2$ and exchange w

$$\rightarrow \begin{pmatrix} 1 & 1 & 1 & 2 & | & 6 \\ 0 & 1 & 1 & 1 & | & 0 \\ 0 & 0 & 3 & -7 & | & -11 \\ 0 & 0 & 1 & 1 & | & 3 \end{pmatrix} \xrightarrow{-3} \begin{pmatrix} 1 & 1 & 1 & 2 & | & 6 \\ 0 & 1 & 1 & 1 & | & 0 \\ 0 & 0 & 1 & 1 & | & 3 \\ 0 & 0 & 3 & -7 & | & -11 \end{pmatrix} \rightarrow$$

$$\rightarrow \begin{pmatrix} 2 & | & 6 \\ 0 & 0 & | & 3 \\ 0 & 0 & 0 & -10 & | & 20 \end{pmatrix} \rightarrow \text{triangular system solve by back substitution}$$

$$\begin{aligned} x_1 + x_2 + x_3 + 2x_4 &= 6 \\ x_2 + x_3 + x_4 &= 0 \Rightarrow x_2 = -x_3 - x_4 \\ x_3 + x_4 &= 3 \Rightarrow x_3 = 3 - x_4 \\ -10x_4 &= 20 \Rightarrow x_4 = -2 \end{aligned}$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 4 \\ -3 \\ 1 \\ -2 \end{pmatrix}$$