

(6d)
$$\begin{cases} x_1 + x_2 + x_3 + x_4 = 0 \\ 2x_1 + x_2 - x_3 + 3x_4 = 0 \\ x_1 - 2x_2 + x_3 + x_4 = 0 \end{cases} \Rightarrow \begin{pmatrix} 1 & 1 & 1 & 1 \\ 2 & 1 & -1 & 3 \\ -1 & 1 & -2 & 1 \end{pmatrix} \quad \begin{array}{l} \text{(homogeneous,} \\ \text{so skip} \\ \text{rhs. column)} \end{array}$$

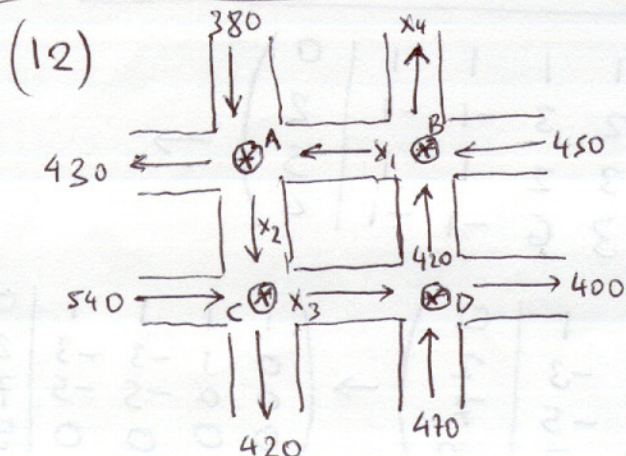
$$\rightarrow \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & -1 & -3 & 1 \\ -3 & 0 & -3 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & -1 & -3 & 1 \\ 0 & 0 & -9 & -3 \end{pmatrix} \xrightarrow{\substack{\text{scale} \\ *(-1) \\ *1/9}} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & -1 \\ 0 & 0 & 1 & -1/3 \end{pmatrix}$$

Note: here, I scale the equations like the text

$$\rightarrow \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & -1 \\ 0 & 0 & 1 & -1/3 \end{pmatrix} \xrightarrow{-1} \begin{pmatrix} 1 & 1 & 0 & 4/3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1/3 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 0 & 4/3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1/3 \end{pmatrix}$$

free

$$\Rightarrow \begin{cases} x_1 = -4/3 x_4 \\ x_2 = 0 \\ x_3 = 1/3 x_4 \end{cases} \quad x_4 \text{ is free or } \alpha \begin{pmatrix} -4/3 \\ 0 \\ 1/3 \\ 1 \end{pmatrix}, \alpha \text{ arbitrary}$$



At (A):
$$\begin{cases} x_1 + 380 = x_2 + 430 \\ 450 + 420 = x_1 + x_4 \\ x_2 + 540 = x_3 + 420 \\ x_3 + 470 = 420 + 400 \end{cases}$$

$$\begin{cases} x_1 - x_2 = 50 \\ x_1 + x_4 = 870 \\ x_2 - x_3 = -120 \\ x_3 = 350 \end{cases} \Rightarrow \begin{pmatrix} 1 & -1 & 0 & 0 & 50 \\ 1 & 0 & 0 & 1 & 870 \\ 0 & 1 & -1 & 0 & -120 \\ 0 & 0 & 1 & 0 & 350 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -1 & 0 & 0 & 50 \\ 0 & 1 & 0 & 1 & 820 \\ 0 & 1 & -1 & 0 & -120 \\ 0 & 0 & 1 & 0 & 350 \end{pmatrix} \xrightarrow{-1} \begin{pmatrix} 1 & -1 & 0 & 0 & 50 \\ 0 & 1 & 0 & 1 & 820 \\ 0 & 0 & 1 & -1 & -120 \\ 0 & 0 & 1 & 0 & 350 \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} 1 & -1 & 0 & 0 & 50 \\ 0 & 1 & 0 & 1 & 820 \\ 0 & 0 & -1 & -1 & -940 \\ 0 & 0 & 1 & 0 & 350 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -1 & 0 & 0 & 50 \\ 0 & 1 & 0 & 1 & 820 \\ 0 & 0 & -1 & -1 & -940 \\ 0 & 0 & 0 & -1 & -590 \end{pmatrix} \xrightarrow{\substack{+1 \\ -1}} \begin{pmatrix} 1 & -1 & 0 & 0 & 50 \\ 0 & 1 & 0 & 1 & 820 \\ 0 & 0 & 1 & 1 & 940 \\ 0 & 0 & 0 & 1 & -590 \end{pmatrix} \xrightarrow{\text{(scale)}} \begin{pmatrix} 1 & -1 & 0 & 0 & 50 \\ 0 & 1 & 0 & 1 & 820 \\ 0 & 0 & 1 & 1 & 940 \\ 0 & 0 & 0 & 1 & -590 \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} 1 & -1 & 0 & 0 & 50 \\ 0 & 1 & 0 & 0 & 230 \\ 0 & 0 & 1 & 0 & 350 \\ 0 & 0 & 0 & 1 & -590 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 0 & 0 & 280 \\ 0 & 1 & 0 & 0 & 230 \\ 0 & 0 & 1 & 0 & 350 \\ 0 & 0 & 0 & 1 & -590 \end{pmatrix}$$