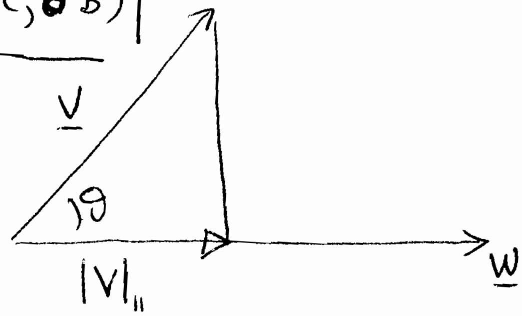
Math. 314 Set 18

p. 206, Sec 5.1 (2a, 3c, 8b)

5.1.2a (Find angle between $\underline{v}, \underline{w}$)

Find scalar/vector projection of $\underline{v}$ onto $\underline{w}$:

$\underline{v} = (2,1,3)^T$, $\underline{w} = (6,3,9)^T$



$$\cos\theta = \frac{v^T w}{\|v\|\,\|w\|}$$

$$|v|_{\parallel} = \|v\|\cos\theta = \frac{v^T w}{\|w\|}$$

$$= \frac{42}{\sqrt{126}} = \frac{42}{3\sqrt{14}} = \boxed{\sqrt{14} = |v|_{\parallel}}$$

scalar projection

$v^T w = 2\cdot 6 + 1\cdot 3 + 3\cdot 9 = 42$

$w^T w = 6^2 + 3^2 + 9^2 = 126$

$\|w\| = \sqrt{126}$

Vector projection:

(i) $p = |v|_{\parallel} \cdot \frac{w}{\|w\|} = \frac{\sqrt{14}}{\sqrt{126}}\, w = \frac{1}{3} w = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$

(using scalar projection)

(ii) (Directly): $p = \frac{v^T w}{w^T w}\, w = \frac{42}{126}\begin{pmatrix} 6 \\ 3 \\ 9 \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 6 \\ 3 \\ 9 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$

5.1.3c Find vector projection p of $\underline{x}$ onto y; verify $x - p \perp p$.

$\underline{x} = (2,4,3)^T$; $y = (1,1,1)^T$; $x^T y = 2+4+3 = 9$

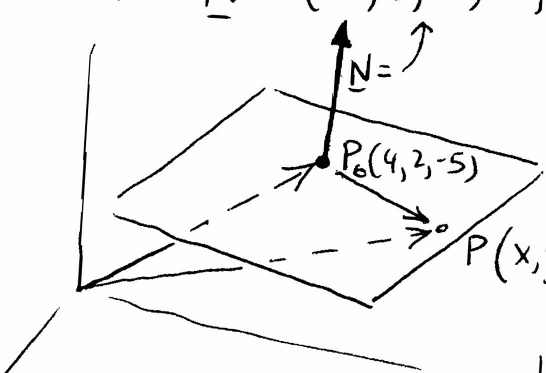
$y^T y = 1+1+1 = 3$

$$p = \frac{x^T y}{y^T y}\, y = \frac{9}{3}\, y = \begin{pmatrix} 3 \\ 3 \\ 3 \end{pmatrix}$$

$$x - p = \begin{pmatrix} 2 \\ 4 \\ 3 \end{pmatrix} - \begin{pmatrix} 3 \\ 3 \\ 3 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} ; \quad (x-p)^T p = (-1, 1, 0)\begin{pmatrix} 3 \\ 3 \\ 3 \end{pmatrix} = -3+3 = 0$$

$\Rightarrow$ orthogonal

5.1.8b $\underline{N} = (-3,6,2)^T$; $P_0 = (4,2,-5)$



Equ. of plane: $\underline{N} \cdot (\vec{P} - \vec{P_0}) = 0$

i.e. $N^T (P - P_0) = 0 \Rightarrow$

$$(-3, 6, 2)\begin{pmatrix} x-4 \\ y-2 \\ z+5 \end{pmatrix} = -3(x-4) + 6(y-2) + 2(z+5) = 0$$

$$\Rightarrow \boxed{-3x + 6y + 2z = -10}$$

Math. 314
Set XIX

p. 215, Sec. 5.2
~~(1,2,4a,7b,15a)~~
(1a,d),2,5,13(a,b)

5.2.1ad Determine a basis for the four subspaces for

(a) $\begin{pmatrix} 3 & 4 \\ 6 & 8 \end{pmatrix}$, (d) $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 2 & 2 \end{pmatrix}$

$$-2\begin{pmatrix} 3 & 4 \\ 6 & 8 \end{pmatrix} \longrightarrow \begin{pmatrix} 3 & 4 \\ 0 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 4/3 \\ 0 & 0 \end{pmatrix}$$

(basic, free*)

$R(A) = \text{span}\left\{\begin{pmatrix} 3 \\ 6 \end{pmatrix}\right\}$; $R(A^T) = \text{span}\left\{\begin{pmatrix} 1 \\ 4/3 \end{pmatrix}\right\}$; $N(A) = \text{span}\left\{\begin{pmatrix} -4/3 \\ 1 \end{pmatrix}\right\}$

$$-3/4\begin{pmatrix} 3 & 6 \\ 4 & 8 \end{pmatrix} \longrightarrow \begin{pmatrix} 3 & 6 \\ 0 & 0 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 2 \\ 0 & 0 \end{pmatrix} ; \quad N(A^T) = \text{span}\left\{\begin{pmatrix} -2 \\ 1 \end{pmatrix}\right\}$$

(d) $$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 2 & 2 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$R(A) = \text{span}\left\{\begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \\ 2 \end{pmatrix}\right\}$; $R(A^T) = \text{span}\left\{\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix}\right\}$

$N(A) = \text{span}\left\{\begin{pmatrix} 0 \\ 0 \\ -1 \\ 1 \end{pmatrix}\right\}$

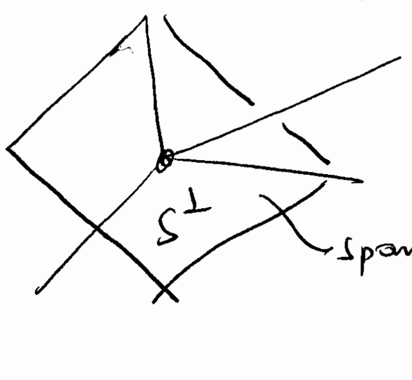
$$A^T = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 2 \\ 0 & 1 & 1 & 2 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} ; \quad N(A^T) = \text{span}\left\{\begin{pmatrix} -1 \\ -1 \\ -1 \\ 1 \end{pmatrix}\right\}$$

5.2.2 $S = \text{span}\left\{\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}\right\}$; $A = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$

(a) $S^\perp$: ~~$\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$~~ $A^T = \begin{pmatrix} 1 & -1 & 1 \end{pmatrix}$:

$$N(A^T) = \text{span}\left\{\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}\right\} = S^\perp$$

(b)



$S = \text{span}\left\{\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}\right\}$ (line)

$\text{span}\left\{\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}\right\}$ (normal plane through $\underline{0}$)

5.3 3.2.5

$P_1 = (1,1,1)$, $P_2 = (2,4,-1)$, $P_3 = (0,-1,5)$

$$\underline{P_1P_2} = \begin{pmatrix} 2-1 \\ 4-1 \\ -1-1 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix} ; \quad \underline{P_1P_3} = \begin{pmatrix} 0-1 \\ -1-1 \\ 5-1 \end{pmatrix} = \begin{pmatrix} -1 \\ -2 \\ 4 \end{pmatrix}$$

$\underline{N}$: find $\mathcal{N}(A^T)$ if $A = \begin{pmatrix} 1 & -1 \\ 3 & -2 \\ -2 & 4 \end{pmatrix}$!

$$A^T = \begin{pmatrix} 1 & 3 & -2 \\ -1 & -2 & 4 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 3 & -2 \\ 0 & 1 & 2 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & -8 \\ 0 & 1 & 2 \end{pmatrix}$$

$$\Rightarrow \quad \underline{N} = \begin{pmatrix} 8 \\ -2 \\ 1 \end{pmatrix} ; \quad \text{Let } P = (x_1, x_2, x_3)$$

Plane: $N^T(\underline{P_1P}) = 0$

$$\Rightarrow \quad \boxed{8(x_1 - 1) - 2(x_2 - 1) + (x_3 - 1) = 0}$$

or $8x_1 - 2x_2 + x_3 = 7$

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(13) (a) if $\underline{x} \in N(A^TA) \Rightarrow A^TA\underline{x} = 0$

either (i) $Ax = 0 \Rightarrow x \in \mathcal{N}(A)$

or (ii) $A^T(Ax) = 0 \Rightarrow (Ax) \in N(A^T)$ } $\Rightarrow Ax \in \{R(A) \cap N(A^T)\}$

but $A\underline{x} \in R(A)$ always

(b) The answer to (a) means that unless $\underline{x} \in N(A)$ we must have $\underline{x} = 0$ (since, by hint given in class only vector common to $R(A)$, $N(A^T)$ is $\underline{x} = 0$

So, $A^TA\underline{x} = 0$ and $A\underline{x} = 0$ have ~~only~~ as solutions (other than $\underline{x}=0$) only vectors $\underline{x} \in N(A)$.

done!