Math 314
Fall '99
Set I

.11 6(e,h), 8(a,b)

(6e)
$$\begin{cases} 2x_1 + x_2 + 3x_3 = 1 \\ 4x_1 + 3x_2 + 5x_3 = 1 \\ 6x_1 + 5x_2 + 5x_3 = -3 \end{cases}$$



$$\left(\begin{array}{ccc|c} 2 & 1 & 3 & 1 \\ 4 & 3 & 5 & 1 \\ 6 & 5 & 5 & -3 \end{array}\right) \begin{matrix} \\ 2 \\ 3 \end{matrix} \longrightarrow \left(\begin{array}{ccc|c} 2 & 1 & 3 & 1 \\ 0 & 1 & -1 & -1 \\ 0 & 2 & -4 & -6 \end{array}\right) \begin{matrix} \\ \\ -2 \end{matrix} \rightarrow \left(\begin{array}{ccc|c} 2 & 1 & 3 & 1 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & -2 & -4 \end{array}\right)$$

Triangular system:

$$2x_1 + x_2 + 3x_3 = 1 \Rightarrow x_1 = \tfrac{1}{2}(1 - x_2 - 3x_3) = \tfrac{1}{2}(-6) = -3$$
$$x_2 - x_3 = -1 \Rightarrow x_2 = -1 + x_3 = 1$$
$$-2x_3 = -4 \Rightarrow x_3 = 2$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -3 \\ 1 \\ 2 \end{pmatrix}$$

(6h)

$$\left(\begin{array}{cccc|c} 0 & 1 & 1 & 1 & 0 \\ 3 & 0 & 3 & -4 & 7 \\ 1 & 1 & 1 & 2 & 6 \\ 2 & 3 & 1 & 3 & 6 \end{array}\right) \rightarrow \begin{matrix} \\ -3 \\ 0 \\ -2 \end{matrix} \left(\begin{array}{cccc|c} 1 & 1 & 1 & 2 & 6 \\ 3 & 0 & 3 & -4 & 7 \\ 0 & 1 & 1 & 1 & 0 \\ 2 & 3 & 1 & 3 & 6 \end{array}\right) \rightarrow$$

$$\rightarrow \left(\begin{array}{cccc|c} 1 & 1 & 1 & 2 & 6 \\ 0 & -3 & 0 & -10 & -11 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & -1 & -1 & -6 \end{array}\right) \rightarrow \begin{matrix} \\ \\ 3 \\ -1 \end{matrix} \left(\begin{array}{cccc|c} 1 & 1 & 1 & 2 & 6 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & -3 & 0 & -10 & -11 \\ 0 & 1 & -1 & -1 & -6 \end{array}\right) \rightarrow \left(\begin{array}{cccc|c} 1 & 1 & 1 & 2 & 6 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 3 & -7 & -11 \\ 0 & 0 & -2 & -2 & -6 \end{array}\right)$$

scale last row by $-1/2$ and exchange w. 3rd

$$\rightarrow \left(\begin{array}{cccc|c} 1 & 1 & 1 & 2 & 6 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 3 & -7 & -11 \\ 0 & 0 & 1 & 1 & 3 \end{array}\right) \rightarrow \begin{matrix} \\ \\ \\ -3 \end{matrix} \left(\begin{array}{cccc|c} 1 & 1 & 1 & 2 & 6 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 3 \\ 0 & 0 & 3 & -7 & -11 \end{array}\right) \rightarrow$$

$$\rightarrow \left(\begin{array}{cccc|c} 1 & 1 & 1 & 2 & 6 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & -10 & -20 \end{array}\right) \rightarrow$$ triangular system; solve by back substitution

$$x_1 + x_2 + x_3 + 2x_4 = 6 \qquad x_1 = 6 - x_2 - x_3 - 2x_4 = 6 + 3 - 1 - 4 = 4$$
$$x_2 + x_3 + x_4 = 0 \Rightarrow x_2 = -x_3 - x_4 = -3$$
$$x_3 + x_4 = 3 \Rightarrow x_3 = 3 - x_4 = 1$$
$$-10x_4 = -20 \Rightarrow x_4 = 2$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 4 \\ -3 \\ 1 \\ 2 \end{pmatrix}$$

8(a,b): combine into one system with two rhs:

$$\begin{matrix} \\ -2 \\ -1 \end{matrix}\left(\begin{array}{ccc|cc} 1 & 2 & -2 & 1 & 9 \\ 2 & 5 & 1 & 9 & 9 \\ 1 & 3 & 4 & 9 & -2 \end{array}\right) \rightarrow \begin{matrix} \\ \\ -1 \end{matrix}\left(\begin{array}{ccc|cc} 1 & 2 & -2 & 1 & 9 \\ 0 & 1 & 5 & 7 & -9 \\ 0 & 1 & 6 & 8 & -11 \end{array}\right) \rightarrow$$

$$\rightarrow \left(\begin{array}{ccc|cc} 1 & 2 & -2 & 1 & 9 \\ 0 & 1 & 5 & 7 & -9 \\ 0 & 0 & 1 & 1 & -2 \end{array}\right)$$

two triangular systems

(a)
$$\begin{aligned} x_1 + 2x_2 - 2x_3 &= 1 \quad &\Rightarrow x_1 = 1 - 2\cdot 2 + 2\cdot 1 = -1 \\ x_2 + 5x_3 &= 7 \quad &\Rightarrow x_2 = 7 - 5 = 2 \\ x_3 &= 1 \end{aligned}$$

(b)
$$\begin{aligned} x_1 + 2x_2 - 2x_3 &= 9 \quad &\Rightarrow x_1 = 9 - 2\cdot(1) - 2\cdot 2 = 3 \\ x_2 + 5x_3 &= -9 \quad &\Rightarrow x_2 = -9 + 5\cdot 2 = 1 \\ x_3 &= -2 \end{aligned}$$

(a) $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}$, (b) $\begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix}$

<u>Note</u>: in problem (6h) used all three elementary row ops. to make sure we were dealing with easy numbers. This kind of trick is important for hand arithmetic but is not necessary if a computer is used and the coefficients are not simple integers.

Math 314
Fall 99
Set II

5(e,g)(j); 6(d), 12 (↳ not dne!)

(5e)
$$\left.\begin{aligned} 2x_1+3x_2+x_3&=1\\ x_1+x_2+x_3&=3\\ 3x_1+4x_2+2x_3&=4\end{aligned}\right\}\Rightarrow$$

$$\begin{matrix} \\ -\tfrac12 \\ -\tfrac32\end{matrix}\left(\begin{array}{ccc|c} 2&3&1&1\\ 1&1&1&3\\ 3&4&2&4\end{array}\right)\longrightarrow \begin{matrix} \\ \\ -1\end{matrix}\left(\begin{array}{ccc|c} 2&3&1&1\\ 0&-\tfrac12&\tfrac12&\tfrac52\\ 0&-\tfrac12&\tfrac12&\tfrac52\end{array}\right)$$

$$\downarrow$$

$$\left(\begin{array}{ccc|c} 2&3&1&1\\ 0&-\tfrac12&\tfrac12&\tfrac52\\ 0&0&0&0\end{array}\right)*2$$

$$\longrightarrow\left(\begin{array}{ccc|c} 2&3&1&1\\ 0&-1&1&5\\ 0&0&0&0\end{array}\right)^{3}\longrightarrow\left(\begin{array}{ccc|c} 2&0&4&16\\ 0&-1&1&5\\ 0&0&0&0\end{array}\right)\begin{matrix}*(\tfrac12)\\ *(-1)\\ \end{matrix}\ \text{scale}$$

Note: here, I only scaled the equations at the very end

$$\downarrow \quad (\text{free})$$

$$\left(\begin{array}{ccc|c} 1&0&2&8\\ 0&1&-1&-5\\ 0&0&0&0\end{array}\right)\Rightarrow \begin{aligned} x_1&=8-2x_3\\ x_2&=-5+x_3\end{aligned}\quad \text{or}\quad \begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=\begin{pmatrix}8\\-5\\0\end{pmatrix}+\alpha\begin{pmatrix}-2\\1\\1\end{pmatrix}$$

(reduced echelon form)

$x_3$ is free
set $x_3=\alpha$

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(5g)
$$\left.\begin{aligned} x_1+x_2+x_3+x_4&=0\\ 2x_1+3x_2-x_3-x_4&=2\\ 3x_1+2x_2+x_3+x_4&=5\\ 3x_1+6x_2-x_3-x_4&=4\end{aligned}\right\}\Rightarrow \begin{matrix} \\ -2\\ -3\\ -3\end{matrix}\left(\begin{array}{cccc|c} 1&1&1&1&0\\ 2&3&-1&-1&2\\ 3&2&1&1&5\\ 3&6&-1&-1&4\end{array}\right)\longrightarrow$$

$$\longrightarrow \begin{matrix} \\ \\ 1\\ -3\end{matrix}\left(\begin{array}{cccc|c} 1&1&1&1&0\\ 0&1&-3&-3&2\\ 0&-1&-2&-2&5\\ 0&3&-4&-4&4\end{array}\right)\longrightarrow \begin{matrix} \\ \\ \\ 1\end{matrix}\left(\begin{array}{cccc|c} 1&1&1&1&0\\ 0&1&-3&-3&2\\ 0&0&-5&-5&7\\ 0&0&+5&5&-2\end{array}\right)\longrightarrow\left(\begin{array}{cccc|c} 1&1&1&1&0\\ 0&1&-3&-3&2\\ 0&0&-5&-5&7\\ 0&0&0&0&5\end{array}\right)$$

Inconsistent

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(5j)
$$\left.\begin{aligned} x_1+2x_2-3x_3+x_4&=1\\ -x_1-x_2+4x_3-x_4&=6\\ -2x_1-4x_2+7x_3-x_4&=1\end{aligned}\right\}\begin{matrix} \\ 1\\ 2\end{matrix}\left(\begin{array}{cccc|c} 1&2&-3&1&1\\ -1&-1&4&-1&6\\ -2&-4&7&-1&1\end{array}\right)\rightarrow\left(\begin{array}{cccc|c} 1&2&-3&1&1\\ 0&1&1&0&7\\ 0&0&1&1&3\end{array}\right)\begin{matrix}3\\ -1\\ \end{matrix}$$

$$\longrightarrow\left(\begin{array}{cccc|c} 1&2&0&4&10\\ 0&1&0&-1&4\\ 0&0&1&1&3\end{array}\right)\rightarrow\left(\begin{array}{cccc|c} 1&0&0&6&2\\ 0&1&0&-1&4\\ 0&0&1&1&3\end{array}\right)$$

(free)

reduced echelon form

$$\left.\begin{aligned} x_1&=2-6x_4\\ x_2&=4+x_4\\ x_3&=3-x_4\end{aligned}\right\}\ x_4\ \text{free}$$

or (set $x_4=\alpha$) ↳ arbitrary

$$\begin{pmatrix}x_1\\x_2\\x_3\\x_4\end{pmatrix}=\begin{pmatrix}2\\4\\3\\0\end{pmatrix}+\alpha\begin{pmatrix}-6\\1\\-1\\1\end{pmatrix}$$

(6d)

$$\left.\begin{aligned} x_1 + x_2 + x_3 + x_4 &= 0 \\ 2x_1 + x_2 - x_3 + 3x_4 &= 0 \\ x_1 - 2x_2 + x_3 + x_4 &= 0 \end{aligned}\right\} \Rightarrow \begin{matrix} \\ -2 \\ -1 \end{matrix}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 2 & 1 & -1 & 3 \\ 1 & -2 & 1 & 1 \end{pmatrix}$$

(homogeneous, so skip rhs. column)

$$\to \begin{matrix} \\ \\ -3 \end{matrix}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & -1 & -3 & 1 \\ 0 & -3 & 0 & 0 \end{pmatrix} \to \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & -1 & -3 & 1 \\ 0 & 0 & 9 & -3 \end{pmatrix}\begin{matrix} \\ *(-1) \\ *1/9 \end{matrix} \xrightarrow{\text{scale}}$$

(Note: here, I scale the equation like the text)

$$\to \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & -1 \\ 0 & 0 & 1 & -1/3 \end{pmatrix}\begin{matrix} -1 \\ -3 \\ \phantom{0} \end{matrix} \to \begin{pmatrix} 1 & 1 & 0 & 4/3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1/3 \end{pmatrix}\begin{matrix} -1 \\ \\ \phantom{0} \end{matrix} \to \begin{pmatrix} 1 & 0 & 0 & 4/3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1/3 \end{pmatrix}$$

(4th column: free)

$$\Rightarrow \left.\begin{aligned} x_1 &= -4/3\, x_4 \\ x_2 &= 0 \\ x_3 &= 1/3\, x_4 \end{aligned}\right\} \quad x_4 \text{ is free} \quad \text{or} \quad \alpha\begin{pmatrix} -4/3 \\ 0 \\ 1/3 \\ 1 \end{pmatrix}, \ \alpha \text{ arbitrary}$$

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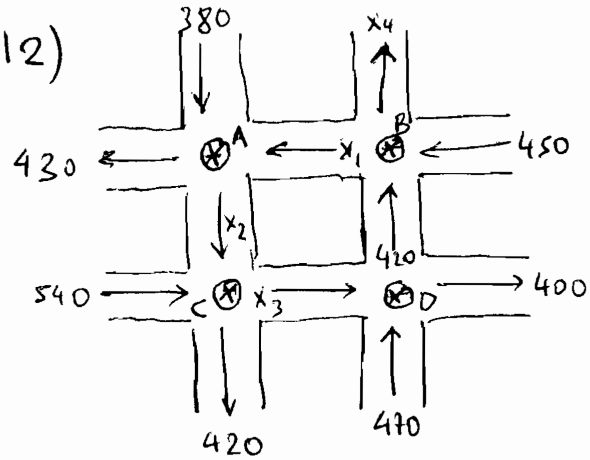
(12)




| | In | | Out |
|---|---|---|---|
| At Ⓐ: | $x_1 + 380$ | $=$ | $x_2 + 430$ |
| Ⓑ | $450 + 420$ | $=$ | $x_1 + x_4$ |
| Ⓒ | $x_2 + 540$ | $=$ | $x_3 + 420$ |
| Ⓓ | $x_3 + 470$ | $=$ | $420 + 400$ |

$$\left.\begin{aligned} x_1 - x_2 &= 50 \\ x_1 + x_4 &= 870 \\ x_2 - x_3 &= -120 \\ x_3 &= 350 \end{aligned}\right\} \Rightarrow \begin{matrix} -1 \\ \\ \\ \phantom{0} \end{matrix}\left(\begin{array}{cccc|c} 1 & -1 & 0 & 0 & 50 \\ 1 & 0 & 0 & 1 & 870 \\ 0 & 1 & -1 & 0 & -120 \\ 0 & 0 & 1 & 0 & 350 \end{array}\right) \to \begin{matrix} \\ -1 \\ \\ \phantom{0} \end{matrix}\left(\begin{array}{cccc|c} 1 & -1 & 0 & 0 & 50 \\ 0 & 1 & 0 & 1 & 820 \\ 0 & 1 & -1 & 0 & -120 \\ 0 & 0 & 1 & 0 & 350 \end{array}\right)$$

$$\to \left(\begin{array}{cccc|c} 1 & -1 & 0 & 0 & 50 \\ 0 & 1 & 0 & 1 & 820 \\ 0 & 0 & -1 & -1 & -940 \\ 0 & 0 & 1 & 0 & 350 \end{array}\right) \to \left(\begin{array}{cccc|c} 1 & -1 & 0 & 0 & 50 \\ 0 & 1 & 0 & 1 & 820 \\ 0 & 0 & -1 & -1 & -940 \\ 0 & 0 & 0 & -1 & -590 \end{array}\right)\begin{matrix} +1 \\ -1 \\ \\ \phantom{0} \end{matrix} \to \left(\begin{array}{cccc|c} 1 & -1 & 0 & 0 & 50 \\ 0 & 1 & 0 & 1 & 820 \\ 0 & 0 & 1 & 1 & 940 \\ 0 & 0 & 0 & 1 & 590 \end{array}\right)$$

(scale)↑

$$\to \left(\begin{array}{cccc|c} 1 & -1 & 0 & 0 & 50 \\ 0 & 1 & 0 & 0 & 230 \\ 0 & 0 & 1 & 0 & 350 \\ 0 & 0 & 0 & 1 & 590 \end{array}\right)\begin{matrix} 1 \\ \\ \\ \phantom{0} \end{matrix} \to \left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 280 \\ 0 & 1 & 0 & 0 & 230 \\ 0 & 0 & 1 & 0 & 350 \\ 0 & 0 & 0 & 1 & 590 \end{array}\right)$$

Math. 314
Fall 99
Set III

10, 13(abc), 23(ab)

(10) $A = \begin{pmatrix} 1/2 & -1/2 \\ -1/2 & 1/2 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} = \frac{1}{2} B$

with $B = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$ ; so $A^n = \left(\frac{1}{2}\right)^n B^n$

$$B^2 = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} -2 & -2 \\ -2 & -2 \end{pmatrix} = -2\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

$$B^3 = -2\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} = -2\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$ ; after that $B^k = 0,\ k \geq 3$

So : $A = \frac{1}{2}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$ ; $A^2 = -\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ ; $A^k = 0,\ k = 3, 4, \ldots$

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(13 a)-b-c) $A = \begin{pmatrix} 1 & 2 \\ 1 & -2 \end{pmatrix}$ ; $\underline{a}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $\underline{a}_2 = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$ ;

(a) Want $x_1\begin{pmatrix} 1 \\ 1 \end{pmatrix} + x_2\begin{pmatrix} 2 \\ -2 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} = \underline{b}$ ; clearly $2\begin{pmatrix} 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 2 \\ -2 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$

so $x_1 = 2$, $x_2 = 1$.

(b) Now $x_1\underline{a}_1 + x_2\underline{a}_2 = \underline{b} \Rightarrow \left.\begin{matrix} x_1 + 2x_2 = 4 \\ x_1 - 2x_2 = 0 \end{matrix}\right\}$ we found $x_1 = 2$, $x_2 = 1$

Solving as a system: $\left(\begin{array}{cc|c} 1 & 2 & 4 \\ 1 & -2 & 0 \end{array}\right) \rightarrow \left(\begin{array}{cc|c} 1 & 2 & 4 \\ 0 & -4 & -4 \end{array}\right) \rightarrow$ (−1)

$\rightarrow \left(\begin{array}{cc|c} 1 & 2 & 4 \\ 0 & 1 & 1 \end{array}\right) \rightarrow \left(\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 1 \end{array}\right)$ consistent, no free variables

$\therefore \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ is the unique solution.

(c) Now the system is $\left(\begin{array}{cc|c} 1 & 2 & -3 \\ 1 & -2 & -2 \end{array}\right)$ (−1)

we expect a unique solution since matrix reduces to echelon form w/o free vars

$$\longrightarrow \left(\begin{array}{cc|c} 1 & 2 & -3 \\ 0 & -4 & 1 \end{array}\right) \rightarrow \left(\begin{array}{cc|c} 1 & 2 & -3 \\ 0 & 1 & -1/4 \end{array}\right) \rightarrow \left(\begin{array}{cc|c} 1 & 0 & -5/2 \\ 0 & 1 & -1/4 \end{array}\right)$$

i.e. $-5/2\begin{pmatrix} 1 \\ 1 \end{pmatrix} - 1/4\begin{pmatrix} 2 \\ -2 \end{pmatrix} = \begin{pmatrix} -3 \\ -2 \end{pmatrix}$

(25)(a) $A$ is $m \times n$, $A'$ is $n \times m$

So: $A^T A$ is $(n \times m) \times (m \times n)$ i.e. $n \times n$

$A A^T$ is $(m \times n) \times (n \times m)$ or $(m \times m)$

(b) A matrix is symmetric if $A = A^T$; then it must be square (i.e. $m = n$)

ules: We have $(A^T)^T = A$ (rule 1, p.50); $(AB)^T = B^T A^T$ (rule 4, p.50)

So ~~Let A / B = A^T?~~ we must show that

$(A^T A)^T = A^T A$ ; we let $A^T = B$

But $(A^T A)^T = (BA)^T = A^T B^T = A^T (A^T)^T = A^T A$

So $A^T A$ is equal to its transpose, and therefore it is symmetric.

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Set IV

9a) $A = \begin{pmatrix} 1 & 0 & 1 \\ 3 & 3 & 4 \\ 2 & 2 & 3 \end{pmatrix}$ ; verify $A^{-1} = \begin{pmatrix} 1 & 2 & -3 \\ -1 & 1 & -1 \\ 0 & -2 & 3 \end{pmatrix}$

$$\begin{pmatrix} 1 & 0 & 1 \\ 3 & 3 & 4 \\ 2 & 2 & 3 \end{pmatrix}\begin{pmatrix} 1 & 2 & -3 \\ -1 & 1 & -1 \\ 0 & -2 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(b) $A\underline{x} = b \Rightarrow \underline{x} = A^{-1}\underline{b}$

(i) $\underline{b} = (1,1,1)^T \Rightarrow \underline{x} = \begin{pmatrix} 1 & 2 & -3 \\ -1 & 1 & -1 \\ 0 & -2 & 3 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1+2-3 \\ -1+1-1 \\ 0-2+3 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$

(ii) $\underline{b} = (1,2,3)^T \Rightarrow \underline{x} = \begin{pmatrix} 1 & 2 & -3 \\ -1 & 1 & -1 \\ 0 & -2 & 3 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 1+4-9 \\ -1+2-3 \\ 0-4+9 \end{pmatrix} = \begin{pmatrix} -4 \\ -2 \\ 5 \end{pmatrix}$

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p.66, 1.4.10 (b,f,h) Find inverses

(b) $-\frac{1}{2}$ $\left(\begin{array}{cc|cc} 2 & 5 & 1 & 0 \\ 1 & 3 & 0 & 1 \end{array}\right) \to \left(\begin{array}{cc|cc} 2 & 5 & 1 & 0 \\ 0 & -7 & -2 & 1 \end{array}\right) \to \left(\begin{array}{cc|cc} 1 & 5/2 & 1/2 & 0 \\ 0 & 1 & 2/7 & [illegible] \end{array}\right) -5/2 \to \left(\begin{array}{cc|cc} 1 & 0 & -3/14 & -5/2 \\ 0 & 1 & 2/7 & 1 \end{array}\right)$

b) $-\frac{1}{2}$ $\left(\begin{array}{cc|cc} 2 & 5 & 1 & 0 \\ 1 & 3 & 0 & 1 \end{array}\right) \to \left(\begin{array}{cc|cc} 2 & 5 & 1 & 0 \\ 0 & 1/2 & -1/2 & 1 \end{array}\right) \to \left(\begin{array}{cc|cc} 1 & 5/2 & 1/2 & 0 \\ 0 & 1 & -1 & 2 \end{array}\right) \xrightarrow{-5/2}$

$\to \left(\begin{array}{cc|cc} 1 & 0 & 3 & -5 \\ 0 & 1 & -1 & 2 \end{array}\right)$ ∴ $\boxed{A^{-1} = \begin{pmatrix} 3 & -5 \\ -1 & 2 \end{pmatrix}}$

f) $-\frac{1}{2}$ $\left(\begin{array}{ccc|ccc} 2 & 0 & 5 & 1 & 0 & 0 \\ 0 & 3 & 0 & 0 & 1 & 0 \\ 1 & 0 & 3 & 0 & 0 & 1 \end{array}\right) \to \left(\begin{array}{ccc|ccc} 2 & 0 & 5 & 1 & 0 & 0 \\ 0 & 3 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1/2 & -1/2 & 0 & 1 \end{array}\right) \to \left(\begin{array}{ccc|ccc} 1 & 0 & 5/2 & 1/2 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1/3 & 0 \\ 0 & 0 & 1 & -1 & 0 & 2 \end{array}\right)$

$\to \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & 0 & -5 \\ 0 & 1 & 0 & 0 & 1/3 & 0 \\ 0 & 0 & 1 & -1 & 0 & 2 \end{array}\right)$ ; $\boxed{A^{-1} = \begin{pmatrix} 3 & 0 & -5 \\ 0 & 1/3 & 0 \\ -1 & 0 & 2 \end{pmatrix}}$

b) $\left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ -1 & 1 & 1 & 0 & 1 & 0 \\ -1 & -2 & -3 & 0 & 0 & 1 \end{array}\right) \to \left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 1 & 0 \\ 0 & -2 & -2 & 1 & 0 & 1 \end{array}\right) \to$

$\to \left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 1 & 0 \\ 0 & 0 & 2 & 3 & 2 & 1 \end{array}\right) \to \left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 1 & 0 \\ 0 & 0 & 1 & 3/2 & 1 & 1/2 \end{array}\right) \to$

$\to \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -1/2 & -1 & -1/2 \\ 0 & 1 & 0 & -2 & -1 & -1 \\ 0 & 0 & 1 & 3/2 & 1 & 1/2 \end{array}\right)$; $A^{-1} = \begin{pmatrix} -1/2 & -1 & -1/2 \\ -2 & -1 & -1 \\ 3/2 & 1 & 1/2 \end{pmatrix}$

Verify: $\begin{pmatrix} 1 & 0 & 1 \\ -1 & 1 & 1 \\ -1 & -2 & -3 \end{pmatrix} \begin{pmatrix} -1/2 & -1 & -1/2 \\ -2 & -1 & -1 \\ 3/2 & 1 & 1/2 \end{pmatrix} =$

$= \begin{pmatrix} -1/2+3/2 & -1+1 & -1/2+1/2 \\ 1/2-2+3/2 & +1-1+1 & 1/2-1+1/2 \\ 1/2+4-9/2 & 1+2-3 & 1/2+2-3/2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ ✓.

(11.) Given $A = \begin{pmatrix} 3 & 1 \\ 5 & 2 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$; Compute $A^{-1}$, use to solve for $X$ : $AX = B$ and $Y$: $YA = B$

i.e. we want
$AX = B \Rightarrow A^{-1}AX = A^{-1}B \Rightarrow X = A^{-1}B$
$YA = B \Rightarrow YAA^{-1} = BA^{-1} \Rightarrow Y = BA^{-1}$

Find $A^{-1}$: $\left(\begin{array}{cc|cc} 3 & 1 & 1 & 0 \\ 5 & 2 & 0 & 1 \end{array}\right) \to \left(\begin{array}{cc|cc} 3 & 1 & 1 & 0 \\ 0 & 1/3 & -5/3 & 1 \end{array}\right) \to \left(\begin{array}{cc|cc} 1 & 1/3 & 1/3 & 0 \\ 0 & 1 & -5 & 3 \end{array}\right) \to$

$\to \left(\begin{array}{cc|cc} 1 & 0 & 2 & -1 \\ 0 & 1 & -5 & 3 \end{array}\right) \Rightarrow A^{-1} = \begin{pmatrix} 2 & -1 \\ -5 & 3 \end{pmatrix}$

$X = A^{-1}B = \begin{pmatrix} 2 & -1 \\ -5 & 3 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 4 & 2 \end{pmatrix}$ | $Y = BA^{-1} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 2 & -1 \\ -5 & 3 \end{pmatrix} = \begin{pmatrix} -8 & 5 \\ -14 & 9 \end{pmatrix}$