Math 314
Fall 99
Set II

5(e,g)(j); 6(d), 12
↳ not due!

(5e)
$$\left.\begin{aligned} 2x_1+3x_2+x_3&=1\\ x_1+x_2+x_3&=3\\ 3x_1+4x_2+2x_3&=4\end{aligned}\right\}\rightarrow$$

$$\begin{matrix}\\ -\frac{1}{2}\\ -\frac{3}{2}\end{matrix}\left(\begin{array}{ccc|c}2&3&1&1\\1&1&1&3\\3&4&2&4\end{array}\right)\rightarrow\left(\begin{array}{ccc|c}2&3&1&1\\0&-\frac{1}{2}&\frac{1}{2}&\frac{5}{2}\\0&-\frac{1}{2}&\frac{1}{2}&\frac{5}{2}\end{array}\right)\begin{matrix}\\\\ -1\end{matrix}$$

$$\downarrow$$

$$\left(\begin{array}{ccc|c}2&3&1&1\\0&-\frac{1}{2}&\frac{1}{2}&\frac{5}{2}\\0&0&0&0\end{array}\right)\begin{matrix}\\\\ *2\end{matrix}\leftarrow\left(\begin{array}{ccc|c}2&3&1&1\\0&-1&1&5\\0&0&0&0\end{array}\right)^{3}\leftarrow\left(\begin{array}{ccc|c}2&0&4&16\\0&-1&1&5\\0&0&0&0\end{array}\right)\begin{matrix}*(\frac{1}{2})\\ *(-1)\\\\\end{matrix}$$

scale

Note: here, I only scaled the equations at the very end

$$\downarrow\ \text{(free)}$$

$$\left(\begin{array}{ccc|c}1&0&2&8\\0&1&-1&-5\\0&0&0&0\end{array}\right)\Rightarrow\begin{aligned}x_1&=8-2x_3\\x_2&=-5+x_3\\x_3&\text{ is free}\\ \text{set } x_3&=\alpha\end{aligned}\qquad\text{or}\quad\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=\begin{pmatrix}8\\-5\\0\end{pmatrix}+\alpha\begin{pmatrix}-2\\1\\1\end{pmatrix}$$

(reduced echelon form)

(5g)
$$\left.\begin{aligned} x_1+x_2+x_3+x_4&=0\\ 2x_1+3x_2-x_3-x_4&=2\\ 3x_1+2x_2+x_3+x_4&=5\\ 3x_1+6x_2-x_3-x_4&=4\end{aligned}\right\}\Rightarrow\begin{matrix}\\-2\\-3\\-3\end{matrix}\left(\begin{array}{cccc|c}1&1&1&1&0\\2&3&-1&-1&2\\3&2&1&1&5\\3&6&-1&-1&4\end{array}\right)\rightarrow$$

$$\rightarrow\begin{matrix}\\1\\-3\end{matrix}\left(\begin{array}{cccc|c}1&1&1&1&0\\0&1&-3&-3&2\\0&-1&-2&-2&5\\0&3&-4&-4&4\end{array}\right)\rightarrow\begin{matrix}\\\\1\end{matrix}\left(\begin{array}{cccc|c}1&1&1&1&0\\0&1&-3&-3&2\\0&0&-5&-5&7\\0&0&+5&5&-2\end{array}\right)\rightarrow\left(\begin{array}{cccc|c}1&1&1&1&0\\0&1&-3&-3&2\\0&0&-5&-5&7\\0&0&0&0&5\end{array}\right)$$

Inconsistent

(5j)
$$\left.\begin{aligned} 2x_1+2x_2-3x_3+x_4&=1\\ -x_1-x_2+4x_3-x_4&=6\\ -2x_1-4x_2+7x_3-x_4&=1\end{aligned}\right\}\begin{matrix}1\\2\\\\\end{matrix}\left(\begin{array}{cccc|c}1&2&-3&1&1\\-1&-1&4&-1&6\\-2&-4&7&-1&1\end{array}\right)\rightarrow\left(\begin{array}{cccc|c}1&2&-3&1&1\\0&1&1&0&7\\0&0&1&1&3\end{array}\right)$$

$$\rightarrow\left(\begin{array}{cccc|c}1&2&0&4&10\\0&1&0&-1&4\\0&0&1&1&3\end{array}\right)\rightarrow\left(\begin{array}{cccc|c}1&0&0&6&2\\0&1&0&-1&4\\0&0&1&1&3\end{array}\right)$$

(free)

reduced echelon form

$$\left.\begin{aligned}x_1&=2-6x_4\\x_2&=4+x_4\\x_3&=3-x_4\end{aligned}\right\}\ x_4\text{ free}$$

or (set $x_4=\alpha$)

$$\begin{pmatrix}x_1\\x_2\\x_3\\x_4\end{pmatrix}=\begin{pmatrix}2\\4\\3\\0\end{pmatrix}+\alpha\begin{pmatrix}-6\\1\\-1\\1\end{pmatrix}$$

↳ arbitrary