314 '03-MIDTERM 1

Name:____

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1 $< 30pts >$

Mark each statement as true or false. Justify your answers. In (a-f), the vectors $\mathbf{v}_1, \ldots, \mathbf{v}_p$ are vectors in a non-zero, finite dimensional vector space V, and $S = \{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$.

a [ ] The set of all linear combinations of the $\mathbf{v}_1, \ldots, \mathbf{v}_p$ is a vector space.

b [ ] If $\{\mathbf{v}_1, \ldots, \mathbf{v}_{p-1}\}$ spans V then S spans V.

c [ ] If $\{\mathbf{v}_1, \ldots, \mathbf{v}_{p-1}\}$ is linearly independent, then so is S.

d [ ] If S is linearly independent, then S is a basis of V.

e [ ] If span$S = V$ then some subset of S is a basis of V.

f [ ] If dim$V = p$ and span$S = V$ then S is a basis of V.

g [ ] An arbitrary plane in $\mathbf{R}^3$ is a two-dimensional subspace.

h [ ] Row operaqtions on a matrix A can change the null space $\mathcal{N}(A)$.

i [ ] The rank of a matrix equals the number of nonzero rows.

j [ ] If an $m \times n$ matrix A is row equivalent to an upper echelon matrix U, and if U has k nonzero rows, then the dimension of the null space of A, $\mathcal{N}(A)$, is equal to $m - k$.

2 $< 15pts >$

Given the matrix

$$A = \begin{bmatrix} 1 & -3 & 4 & -1 & 9 \\ -2 & 6 & -6 & -1 & -10 \\ -3 & 9 & -6 & -6 & -3 \\ 3 & -9 & 4 & 9 & 0 \end{bmatrix}$$

Find bases for the null space $\mathcal{N}(A)$, column space $\mathcal{R}(A)$ and row space $\mathcal{R}(A^T)$. Determine the rank of A and the dimensions of the above spaces.

Solution

3 $< 10pts >$

Let

$$\mathbf{v}_1 = \begin{bmatrix} 3 \\ -6 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} 3 \\ -12 \\ 7 \end{bmatrix}.$$

Let $H = \text{span}\{\mathbf{v}_1, \mathbf{v}_2\}$, then $\mathcal{B} = \{\mathbf{v}_1, \mathbf{v}_2\}$ is a basis for H.

a Is $\mathbf{x} \in H$?

b If the answer to part (a) is yes, find the coordinates of $\mathbf{x}$ in the basis $\mathcal{B}$.

4 $< 15pts >$

By doing **as few calculations as possible** determine which of the following matrices is invertible. Justify your answers.

a

$$\begin{bmatrix} 1 & 3 & 0 & -1 \\ 0 & 1 & -2 & -1 \\ -2 & -6 & 3 & 2 \\ 3 & 5 & 8 & -3 \end{bmatrix}$$

b

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 5 & 6 & 7 \\ 0 & 0 & 8 & 9 \\ 0 & 0 & 0 & 10 \end{bmatrix}$$

c

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 5 & 0 & 0 \\ 3 & 6 & 8 & 0 \\ 4 & 7 & 9 & 10 \end{bmatrix}$$

d

$$\begin{bmatrix} 1 & 3 & -5 \\ 0 & 2 & -3 \\ 0 & -4 & 7 \\ -1 & 5 & -8 \end{bmatrix}$$

5 $< 10pts >$

Find the LU factorization and evaluate the determinant of the matrix

$$A = \begin{pmatrix} 2 & -2 & 4 \\ 1 & -3 & 1 \\ 3 & 7 & 5 \end{pmatrix}$$

6 $< 10pts >$

Find bases for the following sets of vectors:

a All vectors of the form

$$\begin{bmatrix} a & -2b & +c \\ 2a & +5b & -8c \\ -a & -4b & +7c \\ 3a & +b & +c \end{bmatrix}$$

b The vectors in $\mathbf{R}^3$ in the plane $x + 2y + z = 0$.

7 $< 10pts >$

Solve the system $AX = B$ where

$$A = \begin{bmatrix} 1 & 3 & 8 \\ 2 & 4 & 11 \\ 1 & 2 & 5 \end{bmatrix}, \ B = \begin{bmatrix} -3 & 5 \\ 1 & 5 \\ 3 & 4 \end{bmatrix}.$$

(Hint: X is a 3×2 matrix.)