314 '03-QUIZ 3

Name:____

September 22, 2003

1 $< 12pts >$

Find the inverse of the matrix

$$A = \begin{pmatrix} -1 & -3 & -3 \\ 2 & 6 & 1 \\ 3 & 8 & 3 \end{pmatrix}$$

Solution

$$\begin{array}{c} \\ 2 \\ 3 \end{array} \left(\begin{array}{rrr|rrr} -1 & -3 & -3 & 1 & 0 & 0 \\ 2 & 6 & 1 & 0 & 1 & 0 \\ 3 & 8 & 3 & 0 & 0 & 1 \end{array}\right) \to \left(\begin{array}{rrr|rrr} -1 & -3 & -3 & 1 & 0 & 0 \\ 0 & 0 & -5 & 2 & 1 & 0 \\ 0 & -1 & -6 & 3 & 0 & 1 \end{array}\right)$$

Now permute rows 2 and 3:

$$\to \begin{array}{r} -1* \\ -1* \\ -1/5* \end{array} \left(\begin{array}{rrr|rrr} -1 & -3 & -3 & 1 & 0 & 0 \\ 0 & -1 & -6 & 3 & 0 & 1 \\ 0 & 0 & -5 & 2 & 1 & 0 \end{array}\right) \to \begin{array}{r} -3 \\ -6 \\ \\ \end{array} \left(\begin{array}{rrr|rrr} 1 & 3 & 3 & -1 & 0 & 0 \\ 0 & 1 & 6 & -3 & 0 & -1 \\ 0 & 0 & 1 & -2/5 & -1/5 & 0 \end{array}\right)$$

$$\to \begin{array}{r} -3 \\ \\ \\ \end{array} \left(\begin{array}{rrr|rrr} 1 & 3 & 0 & 1/5 & 3/5 & 0 \\ 0 & 1 & 0 & -3/5 & 6/5 & -1 \\ 0 & 0 & 1 & -2/5 & -1/5 & 0 \end{array}\right) \to \left(\begin{array}{rrr|rrr} 1 & 0 & 0 & 2 & -3 & 3 \\ 0 & 1 & 0 & -3/5 & 6/5 & -1 \\ 0 & 0 & 1 & -2/5 & -1/5 & 0 \end{array}\right)$$

$$A^{-1} = \begin{pmatrix} 2 & -3 & 3 \\ -3/5 & 6/5 & -1 \\ -2/5 & -1/5 & 0 \end{pmatrix}$$

2 $< 13pts >$

Find the LU factorization and evaluate the determinant of the matrix

$$A = \begin{pmatrix} -2 & 1 & 2 \\ 4 & 1 & -2 \\ -6 & -3 & 4 \end{pmatrix}$$

Solution

$$\begin{array}{c} \\ 2 \\ -3 \end{array} \begin{pmatrix} -2 & 1 & 2 \\ 4 & 1 & -2 \\ -6 & -3 & 4 \end{pmatrix} \to \begin{array}{c} \\ \\ 2 \end{array} \begin{pmatrix} -2 & 1 & 2 \\ 0 & 3 & 2 \\ 0 & 0 & 2 \end{pmatrix} = U$$

Then

$$L = E_1^{-1} E_2^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 3 & -2 & 1 \end{pmatrix}$$

The determinant is simply the product of the diagonal elements of U, that is

$$detA = detU = (-2) * (3) * (2) = -12 \ .$$