314 '03-QUIZ 4

Name:____

September 30, 2003

1 $< 12pts >$

Determine the nullspace of the matrix

$$\begin{pmatrix} 1 & 2 & 3 & 1 \\ 2 & -4 & 6 & -3 \end{pmatrix}$$

Solution

$$\begin{array}{c} \\ -2 \end{array}\begin{pmatrix} 1 & 2 & 3 & 1 \\ 2 & -4 & 6 & -3 \end{pmatrix} \to \begin{array}{c} \\ -(1/8)* \end{array}\begin{pmatrix} 1 & 2 & 3 & 1 \\ 0 & -8 & 0 & -5 \end{pmatrix} \to$$

$$\begin{array}{c} -2 \\ \\ \end{array}\begin{pmatrix} 1 & 2 & 3 & 1 \\ 0 & 1 & 0 & 5/8 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & 3 & -1/4 \\ 0 & 1 & 0 & 5/8 \end{pmatrix}$$

so that x_1, x_2 are basic variables and x_3, x_4 free. To find the null vectors we have:
(a) $x_3 = 1$, $x_4 = 0$ then $x_1 = -3$, $x_2 = 0$.
(a) $x_3 = 0$, $x_4 = 1$ then $x_1 = 1/4$, $x_2 = -5/8$.
so that the two null vectors are:

$$\mathbf{n}_1 = \begin{pmatrix} -3 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \ \mathbf{n}_2 = \begin{pmatrix} 1/4 \\ -5/8 \\ 0 \\ 1 \end{pmatrix}.$$

2 $< 13pts >$

Given

$$\mathbf{x}_1 = \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix}, \quad \mathbf{x}_2 = \begin{pmatrix} -3 \\ -2 \\ 4 \end{pmatrix}, \quad \mathbf{x} = \begin{pmatrix} 9 \\ -5 \\ -2 \end{pmatrix}$$

Is $\mathbf{x} \in \mathrm{Span}(\mathbf{x}_1, \mathbf{x}_2)$? Prove your answer.

Solution

We are looking for constants C_1, C_2 such that

$$C_1\mathbf{x}_1 + C_2\mathbf{x}_2 = \mathbf{x} \ .$$

Set up the augmented matrix

$$\begin{array}{r} \\ 3 \\ -2 \end{array} \begin{pmatrix} 1 & -3 & | & 9 \\ -3 & -2 & | & -5 \\ 2 & 4 & | & -2 \end{pmatrix} \rightarrow \begin{array}{r} \\ \\ 10/11 \end{array} \begin{pmatrix} 1 & -3 & | & 9 \\ 0 & -11 & | & 22 \\ 0 & 10 & | & -20 \end{pmatrix} \rightarrow$$

$$\begin{array}{r} \\ -1/11 \\ \\ \end{array} \begin{pmatrix} 1 & -3 & | & 9 \\ 0 & -11 & | & 22 \\ 0 & 0 & | & 0 \end{pmatrix} \rightarrow \begin{array}{r} 3 \\ \\ \\ \end{array} \begin{pmatrix} 1 & -3 & | & 9 \\ 0 & 1 & | & -2 \\ 0 & 0 & | & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & | & 3 \\ 0 & 1 & | & -2 \\ 0 & 0 & | & 0 \end{pmatrix}$$

so that

$$3\mathbf{x}_1 - 2\mathbf{x}_2 = \mathbf{x} \ .$$