314 '03-QUIZ 6

Name:____

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1 $< 12pts >$

Given

$$\mathbf{v}_1 = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \ , \ \mathbf{v}_2 = \begin{bmatrix} -1 \\ 2 \end{bmatrix} \ , \ S = \begin{bmatrix} 3 & -1 \\ 1 & 3 \end{bmatrix} \ .$$

Find vectors $\mathbf{u}_1$, $\mathbf{u}_2$ so that S is the transition matrix from $[\mathbf{u}_1 \ , \ \mathbf{u}_2]$ to $[\mathbf{v}_1 \ , \ \mathbf{v}_2]$.

Solution

$$S = V^{-1}U \Rightarrow U = VS = \begin{pmatrix} 2 & -1 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} 3 & -1 \\ 1 & 3 \end{pmatrix} = \begin{pmatrix} 5 & -5 \\ 11 & 3 \end{pmatrix}$$

so

$$\mathbf{u}_1 = \begin{bmatrix} 5 \\ 11 \end{bmatrix} \ , \ \mathbf{u}_2 = \begin{bmatrix} -5 \\ 3 \end{bmatrix}$$

2 $< 13pts >$

Find a basis of the row space, column space and null space of

$$A = \begin{bmatrix} 1 & -1 & 0 & 1 & 2 \\ 2 & -1 & 1 & 3 & 1 \\ -1 & 2 & 1 & -1 & -3 \end{bmatrix}$$

and give the dimension of each.

Solution (use reduced row echelon form to find null vectors most easily)

$$\begin{array}{r} \\ -2 \\ 1 \end{array}\begin{bmatrix} 1 & -1 & 0 & 1 & 2 \\ 2 & -1 & 1 & 3 & 1 \\ -1 & 2 & 1 & -1 & -3 \end{bmatrix} \rightarrow \begin{array}{r} \\ \\ -1 \end{array}\begin{bmatrix} 1 & -1 & 0 & 1 & 2 \\ 0 & 1 & 1 & 1 & -3 \\ 0 & 1 & 1 & 0 & -1 \end{bmatrix} \rightarrow$$

$$\begin{array}{r} \\ \\ -1* \end{array}\begin{bmatrix} 1 & -1 & 0 & 1 & 2 \\ 0 & 1 & 1 & 1 & -3 \\ 0 & 0 & 0 & -1 & 2 \end{bmatrix} \rightarrow \begin{array}{r} -1 \\ -1 \\ \\ \end{array}\begin{bmatrix} 1 & -1 & 0 & 1 & 2 \\ 0 & 1 & 1 & 1 & -3 \\ 0 & 0 & 0 & 1 & -2 \end{bmatrix} \rightarrow$$

$$\begin{array}{r} 1 \\ \\ \\ \end{array}\begin{bmatrix} 1 & -1 & 0 & 0 & 4 \\ 0 & 1 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & -2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 1 & 0 & 3 \\ 0 & 1 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & -2 \end{bmatrix}$$

so the pivots are found on columns 1, 2, 4 and so x_3 , x_5 are free variables. Therefore the dimensions of the column and row spaces are equal to 3, with the rows of A giving the basis of the row space, $\mathcal{R}(A^T)$, and columns 1,2, and 4 of A giving the basis of the column space, $\mathcal{R}(A)$. The dimension of the null space is 2. To find the basis of the null space:

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 3 \\ 0 & 1 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = 0 \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_4 \end{bmatrix} = -x_3 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} - x_5 \begin{bmatrix} 3 \\ -1 \\ -2 \end{bmatrix}$$

so that the null vectors (basis of $\mathcal{N}(A)$) are:

$$(x_3 = 1\ ,\ x_5 = 0 :)\ \mathbf{n}_1 = \begin{bmatrix} -1 \\ -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}\ ,\ (x_3 = 0\ ,\ x_5 = 1 :)\ \mathbf{n}_2 = \begin{bmatrix} -3 \\ 1 \\ 0 \\ 2 \\ 1 \end{bmatrix}$$