Math. 316, ODEs
Exam I —- Spring 2004

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NAME: . .

1. (25pts.) Find the inverse Laplace transforms

   (a) (5pts)

   $$F(s) = \frac{1}{(s+1)(s-1)(s+2)}$$

   (b) (5pts)

   $$F(s) = \frac{1}{s^2(s^2+1)}$$

   (c) (5pts)

   $$F(s) = \frac{2s+7}{s^2+4s+13}$$

(d) (5pts)

$$F(s) = \frac{s-1}{s^4 + 3s^3 + 2s^2}$$

(e) (5pts)

$$F(s) = \frac{s^2 + 2s + 4}{(s^2 + 4s + 5)^2}$$

2. (15pts.) Solve the IVP

$$\frac{dx}{dt} = 4x - 3x^2 \ , \ x(0) = 1$$

and sketch the direction field and the solution.

3. (12pts.) Give the general solution for the ODE

$$\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + k^2y = 0$$

and state if the system is overdamped, critically damped, or underdamped when the spring constant k has the values:

(a) $k = 5$.

(b) $k = 3$.

(c) $k = 2$.

4. (14pts.) Solve the IVP

$$\frac{dy}{dt} + \frac{t}{t^2+1}y = t \ , \ y(0) = 1$$

and identify the largest interval in $t_1 < t < t_2$ for which the solution is defined.

5. (6pts.) Compute the Wronskian of the functions $y_1(x) = \sin 2x$ and $y_2(x) = \cos x \sin x$. Are these linearly independent?

6. (14pts.) Solve the IVP using Laplace transforms:

$$\frac{d^2y}{dt^2} - y = e^{-t}\cos t \ , \ y(0) = y'(0) = 0 \ .$$

7. (14pts.) Solve the IVP using Laplace transforms:

$$\frac{d^2y}{dt^2} + 5\frac{dy}{dt} - 6y = e^{-6t} \ , \ y(0) = 0, y'(0) = 1 \ .$$