316-TEST 2

Name:____

May 3, 2004

(1) (15 pts) Solve the initial value problem and sketch the phase plane. Draw in the solution curve corresponding to the given initial conditions.

$$\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 & 4 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}, \begin{pmatrix} x(0) \\ y(0) \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}.$$

(2) (20 pts) Give the type and stability of the critical point at the origin for the following systems:

1.

$$\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 & -1 \\ 3 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}.$$

2.

$$\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & -4 \\ 4 & -7 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}.$$

3.

$$\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 & -2 \\ 4 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}.$$

4.

$$\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 & -5 \\ 1 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}.$$

(3) (20 pts) Rewrite the ODE

$$\frac{d^2x}{dt^2} + x - x^2 = 0$$

as a system of two first-order equations for x and $y = dx/dt$. Find all the equilibrium points of the system, give their type and stability, find the linearized solution and sketch the phase plane near each one. Give a rough drawing of the full phase plane.

(4) (15 pts) Solve the given IVP using Laplace transforms:

$$y'' + 5y' + 6y = -u(t-\pi) + u(t-2\pi) \ , \ y(0) = 0 \ , \ y'(0) = 0 \ .$$

(5) (15 pts) Give a formula for the solution to the IVP using convolutions:

$$y'' - 2y' + 5y = g(t) \ , \ \ y(0) = 0 \ , \ \ y'(0) = 2 \ .$$

(6) (15) Determine the form of the general solution for the following ODE:

(You can use the method of undetermined coefficients and/or Laplace transforms. No need to solve for the coefficients!)

(a) $y'' + 4y = \cos 2x + \cos x$

(b) $y'' + 2y' + 5y = e^x \cos 2x$

(c) $y'' - y = e^x$