316-TEST 2

Name:____

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Instructions: You must work problems 1-3 and either problem 4 or problem 5. All problems are weighted equally.

(1) (25 pts) Solve the initial value problem and sketch the phase plane. Draw in the solution curve corresponding to the given initial conditions.

$$\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}, \begin{pmatrix} x(0) \\ y(0) \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}.$$

(2) (25 pts) Find the eigenvalues and eigenvectors, write the general solution and give the type and stability of the critical point at the origin for the following systems:

1.

$$\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 & 1 \\ 1 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} .$$

2.

$$\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & -5 \\ 1 & -3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} .$$

3.

$$\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} .$$

4.

$$\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ -5 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} .$$

(3) (25 pts) Consider the system of two first-order equations for x, y:

$$\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 - 2y \\ 12 - 3x^2 \end{pmatrix} .$$

Find all the equilibrium points of the system, give their type and stability, find the linearized solution and sketch the phase plane near each one. Give a rough drawing of the full phase plane.

(4)(25pts) Solve the initial value problem for the system

$$\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

with $x(0) = 1$, $y(0) = -1$. You must use the variation of parameters formula:

$$\mathbf{x}(t) = \mathbf{\Phi}(t)\mathbf{\Phi}^{-1}(0)\mathbf{c} + \mathbf{\Phi}(t)\int_{t_0}^{t} \mathbf{\Phi}^{-1}(s)\mathbf{f}(s)ds \ ,$$

where $\mathbf{\Phi}(t)$ is a fundamental matrix for the homogeneous system and $\mathbf{c}$ a constant vector.

(5) (25pts) Determine the form of the general solution for the following ODE:

(You can use the method of undetermined coefficients and/or Laplace transforms. No need to solve for the coefficients!)

(a) $y'' + y = \cos 2x + \sin x$

(b) $y'' + 2y' + y = e^x \cos 2x$

(c) $y'' + 4y = e^{2x}$