# 316-TEST 2 prep

Name:_____

December 6, 2008

| Problem | grade |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| Total | |

**(1) (20 pts)** Find the general solution of the inhomogeneous differential equation (use any method)

$$y'' + y = t(1 + \cos t) \ .$$

**(2) (20 pts)** Find the general solution of the inhomogeneous differential equation (use any method)

$$y'' + 4y' + 4y = t^{-2}e^{-2t} \ .$$

**(3) (20 pts)** Give the inverse Laplace transforms of the following functions:

1.

$$\frac{-s^3 + 2s^2 - 7s + 3}{s^3(s+1)(s-1)^2}$$

2.

$$\frac{s^2 + 3s - 1}{(s-1)(s^2+1)}$$

3.

$$\frac{2s+1}{s^2 + 2s + 2}$$

4.

$$\frac{s^2 + 3s - 1}{(s^2 + 2s + 1)(s^2 + 2s + 2)}$$

**(4) (20 pts)** Solve the initial value problem using Laplace transforms

$$y'' - 2y' + 2y = e^{-t} \; ; \; y(0) = 1 \; ; \; y'(0) = 0 \; .$$

**(5) (20 pts)** Determine all critical points. Find the corresponding linear system near each critical point. Find the eigenvalues and draw conclusions about type and stability of each critical point. Draw as much of the phase portrait as you can, incorporating all of the above information.

$$\begin{aligned} \frac{dx}{dt} &= y + x(1 - x^2 - y^2) \\ \frac{dy}{dt} &= -x + y(1 - x^2 - y^2) \end{aligned}$$