316-QUIZ 11-12

Name:_____

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(**1**) Find the general solution of the system

$$\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -4 & -3 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}.$$

(2) Find the general solution of the system

$$\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -3 & 5 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}.$$

(3) Solve the initial value problem

$$\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 & 4 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}, \begin{pmatrix} x(0) \\ y(0) \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}.$$

(4) Find the general solution of the system

$$\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 0 \\ 2 \end{pmatrix}.$$

You must use the variation of parameters formula:

$$\mathbf{x}_{gen}(t) = \mathbf{X}(t)\mathbf{c} + \mathbf{X}(t)\int_{t_0}^{t} \mathbf{X}^{-1}(s)\mathbf{f}(s)ds \ ,$$

where $\mathbf{X}(t)$ is a fundamental matrix for the homogeneous system and $\mathbf{c}$ a constant vector.