

Problem 12.4.17: Energy

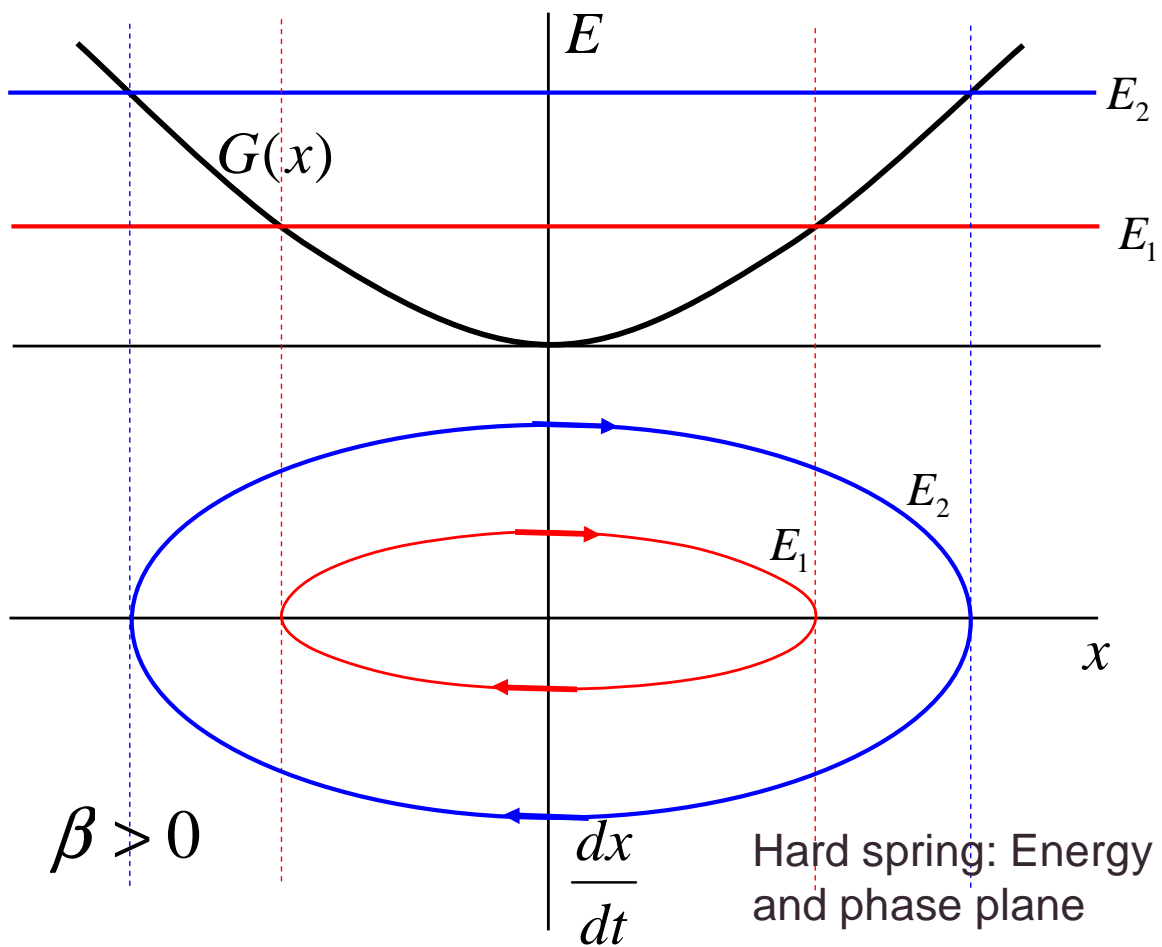
$$\frac{d^2 x}{dt^2} + \alpha x + \beta x^3 = 0 \Rightarrow$$

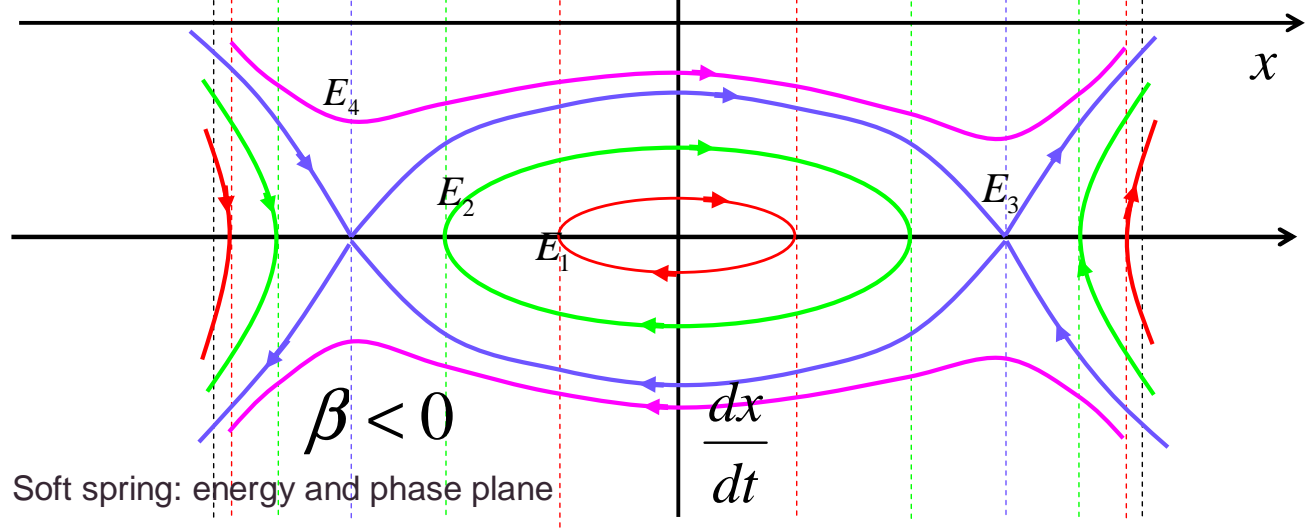
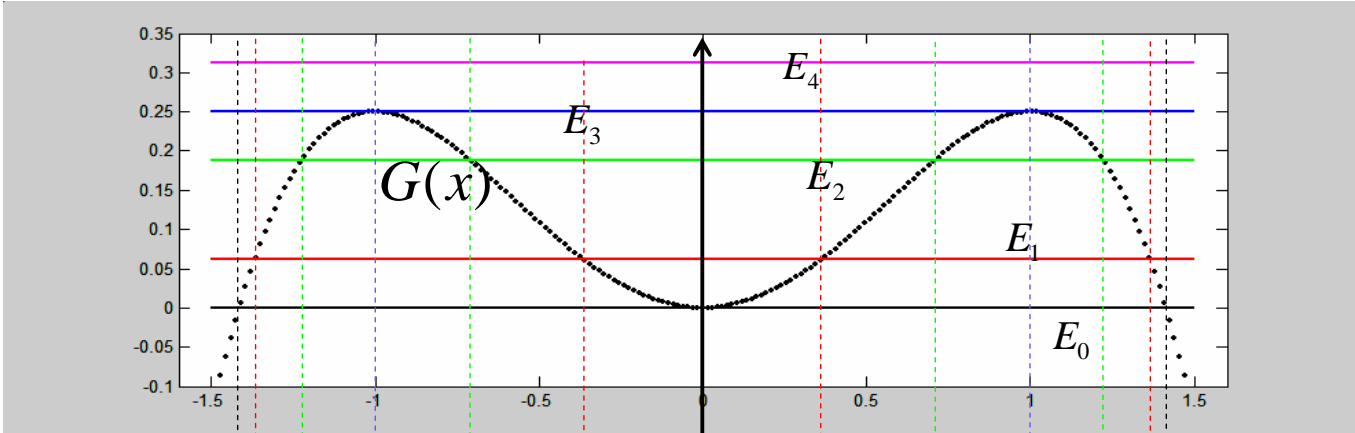
$$\frac{dx}{dt} \frac{d^2 x}{dt^2} + \alpha x \frac{dx}{dt} + \beta x^3 \frac{dx}{dt} = 0 \Rightarrow$$

$$\frac{d}{dt} \left( \frac{1}{2} \left( \frac{dx}{dt} \right)^2 + \frac{\alpha}{2} x^2 + \frac{\beta}{4} x^4 \right) = 0 \Rightarrow$$

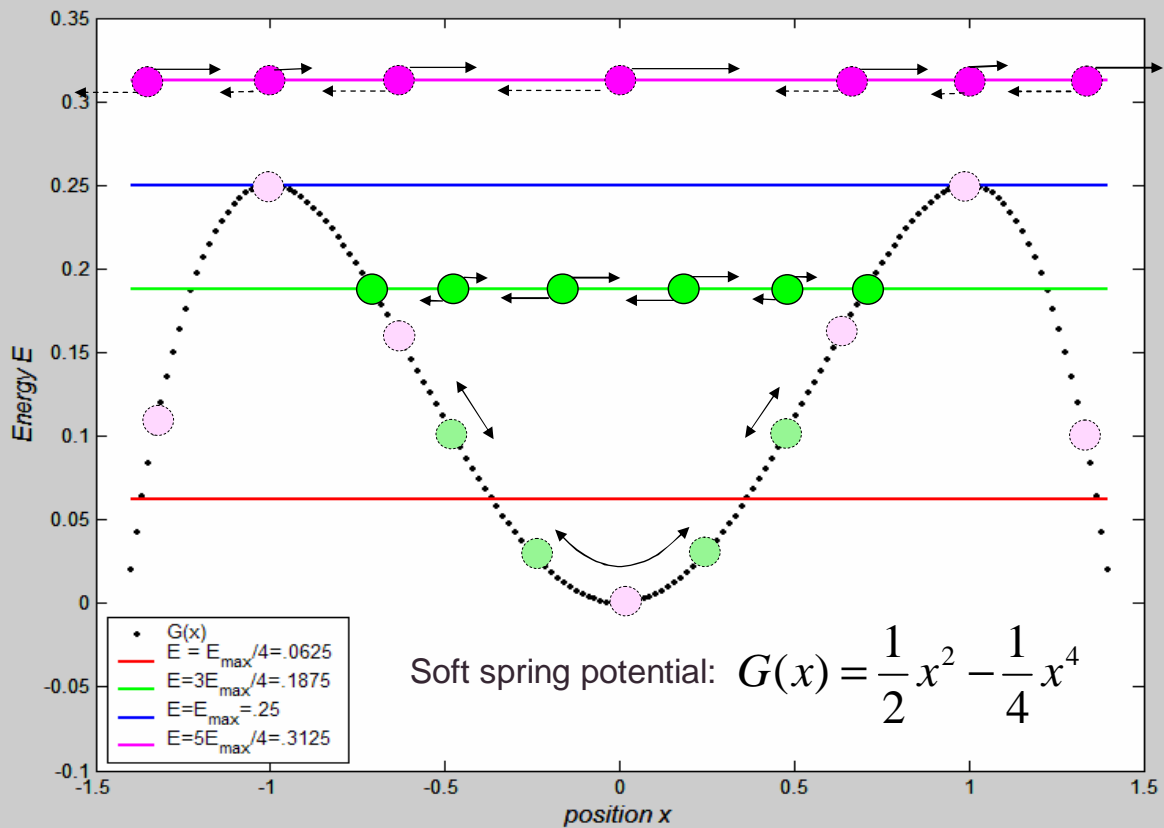
$$\frac{d}{dt} \left( \frac{1}{2} \left( \frac{dx}{dt} \right)^2 + G(x) \right) = 0 \Rightarrow \frac{1}{2} \left( \frac{dx}{dt} \right)^2 + G(x) = E$$

$$G(x) = \frac{\alpha}{2} x^2 + \frac{\beta}{4} x^4$$





Soft spring: energy and phase plane



Motion at two energy levels. The slower particle is inside a potential “well”. The markers are placed at equal time intervals.