Solutions, 316-XXI

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9.2¡1*,2,3*,5,11*¿, 9.3¡3*,4,17*,18¿ CAUTION: there may be errors!!!

1 Problem 9.2.1

Find all solutions using the Gauss-Jordan elimination algorithm:

$$\begin{array}{rcrcrcr} x_1 & + & 2x_2 & + & 2x_3 & = & 6\ , \\ 2x_1 & + & x_2 & + & x_3 & = & 6\ , \\ x_1 & + & x_2 & + & 3x_3 & = & 6\ . \end{array}$$

Solution:

$$\begin{array}{rcrcrcr} x_1 & + & 2x_2 & + & 2x_3 & = & 6\ , \\ 2x_1 & + & x_2 & + & x_3 & = & 6\ , \\ x_1 & + & x_2 & + & 3x_3 & = & 6\ . \end{array}$$

multiply the first row by 2 and subtract from the second row
subtract the first row from the third row:

$$\begin{array}{rcrcrcr} x_1 & + & 2x_2 & + & 2x_3 & = & 6\ , \\ & - & 3x_2 & - & 3x_3 & = & -6\ , \\ & - & x_2 & + & x_3 & = & 0\ . \end{array}$$

Now multiply the third row by 3 and subtract from the second row to get:

$$\begin{array}{rcrcrcr} x_1 & + & 2x_2 & + & 2x_3 & = & 6\ , \\ & & & - & 6x_3 & = & -6\ , \\ & - & x_2 & + & x_3 & = & 0\ . \end{array}$$

Now solve and back-substitute:

$$\begin{aligned} x_3 &= 1 \\ x_2 &= x_3 &&= 1 \\ x_1 &= 6 - 2x_2 - 2x_3 &&= 2\ . \end{aligned}$$

2 Problem 9.2.3

Find all solutions using the Gauss-Jordan elimination algorithm:

$$\begin{array}{rcrcrcr} x_1 & + & x_2 & + & x_3 & = & -3\ , \\ 2x_1 & + & 4x_2 & - & x_3 & = & 0\ , \\ x_1 & + & 3x_2 & - & 2x_3 & = & 3\ . \end{array}$$

Solution:
multiply the first row by 2 and subtract from the second row
subtract the first row from the third row:

$$\begin{array}{rcrcrcr} x_1 & + & x_2 & + & x_3 & = & -3\ , \\ & & 2x_2 & - & 3x_3 & = & 6\ , \\ & & 2x_2 & - & 3x_3 & = & 6\ . \end{array}$$

Notice that the last two rows are now identical; one of them is redundant. We can use x_3 as a free parameter. Then

$$x_2 = \frac{1}{2}(6 + 3x_3) = 3 + \frac{3}{2}x_3$$

and

$$x_1 = -3 - x_2 - x_3 = -3 - \left(3 + \frac{3}{2}x_3\right) - x_3 = -6 - \frac{5}{2}x_3\ .$$

3 Problem 9.2.11

Find all solutions using the Gauss-Jordan elimination algorithm:

$$\begin{array}{rcrcrcr} 2x_1 & & & + & x_3 & = & -1\ , \\ -3x_1 & + & x_2 & + & 4x_3 & = & 1\ , \\ -x_1 & + & x_2 & + & 5x_3 & = & 0\ . \end{array}$$

Solution:
multiply the third row by 2 and add from the first row
multiply the third row by 3 and subtract from the second row

$$\begin{array}{rcrcrcr} & & 2x_2 & + & 11x_3 & = & -1\ , \\ & - & 2x_2 & - & 11x_3 & = & 1\ , \\ -x_1 & + & x_2 & + & 5x_3 & = & 0\ . \end{array}$$

Notice that the first two rows are now identical; one of them is redundant. We can use x_3 as a free parameter. Then

$$x_2 = \frac{1}{2}(-1 - 11x_3) = -\frac{1}{2} - \frac{11}{2}x_3$$

and

$$x_1 = x_2 + 5x_3 = -\frac{1}{2} - \frac{1}{2}x_3\ .$$

4 Problem 9.3.3

Let $\mathbf{A} := \begin{bmatrix} 2 & 4 \\ 1 & 1 \end{bmatrix}$ and $\mathbf{B} := \begin{bmatrix} -1 & 3 \\ 5 & 2 \end{bmatrix}$. Find:

1. **AB**
   Solution:

$$\begin{pmatrix} 2 & 4 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} -1 & 3 \\ 5 & 2 \end{pmatrix} = \begin{pmatrix} 2\cdot(-1)+4\cdot 5 & 2\cdot 3+4\cdot 2 \\ 1\cdot(-1)+1\cdot 5 & 1\cdot 3+1\cdot 2 \end{pmatrix} = \begin{pmatrix} 18 & 14 \\ 4 & 5 \end{pmatrix}$$

2. $\mathbf{A}^2 = \mathbf{AA}$.

$$\begin{pmatrix} 2 & 4 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 2 & 4 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 2\cdot(2)+4\cdot 1 & 2\cdot 4+4\cdot 1 \\ 1\cdot(2)+1\cdot 1 & 1\cdot 4+1\cdot 1 \end{pmatrix} = \begin{pmatrix} 8 & 12 \\ 3 & 5 \end{pmatrix}$$

3. $\mathbf{B}^2 = \mathbf{BB}$.

$$\begin{pmatrix} -1 & 3 \\ 5 & 2 \end{pmatrix} \begin{pmatrix} -1 & 3 \\ 5 & 2 \end{pmatrix} = \begin{pmatrix} -1\cdot(-1)+3\cdot 5 & -1\cdot 3+3\cdot 2 \\ 5\cdot(-1)+2\cdot 5 & 5\cdot 3+2\cdot 2 \end{pmatrix} = \begin{pmatrix} 9 & 3 \\ 5 & 19 \end{pmatrix}$$

5 Problem 9.3.17

FInd the matrix function $\mathbf{X}^{-1}(t)$ whose value at t is the inverse of the matrix $\mathbf{X}(t) := \begin{bmatrix} e^t & e^{4t} \\ e^t & 4e^{4t} \end{bmatrix}$

Solution:

$$\mathbf{X}^{-1} = \frac{1}{e^t 4e^{4t} - e^{4t}e^t} \begin{bmatrix} 4e^{4t} & -e^{4t} \\ -e^t & e^t \end{bmatrix} = \frac{1}{3e^{5t}} \begin{bmatrix} 4e^{4t} & -e^{4t} \\ -e^t & e^t \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 4e^{-t} & -e^{-t} \\ -e^{-4t} & e^{-4t} \end{bmatrix}$$