Help for 316-V, problem 4.1.4

February 5, 2003

1 Problem 4.1.4

Discussion of Resonance: consider the forced/damped system:

$$my'' + by' + ky = \cos \Omega t \ .$$

1. Find the synchronous solution $y(t) = A \cos \Omega t + B \sin \Omega t$
2. Sketch graphs of the coefficients A, B as functions of Ω for $m = 1$, $b = 0.1$, $k = 25$.

Solution:

1. We substitute:

$$\begin{aligned} y(t) &= A \cos \Omega t + B \sin \Omega t \\ y'(t) &= -\Omega A \sin \Omega t + \Omega B \cos \Omega t \\ y''(t) &= -\Omega^2 A \cos \Omega t - \Omega^2 B \sin \Omega t \end{aligned}$$

into the ODE:

$$\begin{aligned} ky &= kA \cos \Omega t + kB \sin \Omega t \\ + &\quad + \\ b\frac{dy}{dt} &= -b\Omega A \sin \Omega t + b\Omega B \cos \Omega t \\ + &\quad + \\ m\frac{d^2y}{dt^2} &= -m\Omega^2 A \cos \Omega t - m\Omega^2 B \sin \Omega t \\ = &\quad = \\ \cos \Omega t &= ((k - m\Omega^2)A + b\Omega B) \cos \Omega t + (-b\Omega A + (k - m\Omega^2)B) \sin \Omega t \end{aligned}$$

We now set the coefficients of $\sin \Omega t$ and $\cos \Omega t$ on both sides of the equation equal to each other, to get the system:

$$\begin{array}{rrcl} (k - m\Omega^2)A & +b\Omega B & = & 1 \\ -b\Omega A & +(k - m\Omega^2)B & = & 0 \end{array}$$

Solving this system we find:

$$A = \frac{k - m\Omega^2}{(k - m\Omega^2)^2 + (b\Omega)^2}$$

$$B = \frac{b\Omega}{(k - m\Omega^2)^2 + (b\Omega)^2}$$

Then, the solution is:

$$y(t) = \frac{(k - m\Omega^2)\sin \Omega t + b\Omega \cos \Omega t}{(k - m\Omega^2)^2 + (b\Omega)^2}$$

2. We now plot A, B as functions of Ω; we give the plot using both Maple and Matlab.

   (a) **MATLAB**:

```
function rescoeff(m,b,k)
% plots the forced vibration coefficients A,B as functions
% of the forcing frequency Omega
Om(1:202) = 0; dOm = 2*sqrt(k/m)/200;
for i = 1:201
a1 = Om(i)*b;
Om2 = Om(i)*Om(i);
a2 = k - m*Om2;
denom = a1*a1 + a2*a2;
A(i) = a2 / denom;
B(i) = a1 / denom;
Om(i+1) = Om(i) + dOm;
end
subplot(2,1,1)
plot(Om(1:201),A,'r-')
title(sprintf('graph of A vs. Omega, with Omega0= %6.2f', sqrt(k/m)))
subplot(2,1,2)
```

plot(Om(1:201),B,'b-')
title(sprintf('graph of B vs. Omega, with Omega0= %6.2f', sqrt(k/m)))

(b) **MAPLE**

```
> restart:
> k:=25;
```

$$k := 25$$

```
> b:=.1;
```

$$b := .1$$

```
> m:=1;
```

$$m := 1$$

```
> Om0 := evalf(sqrt(k/m));
```

$$Om0 := 5.$$

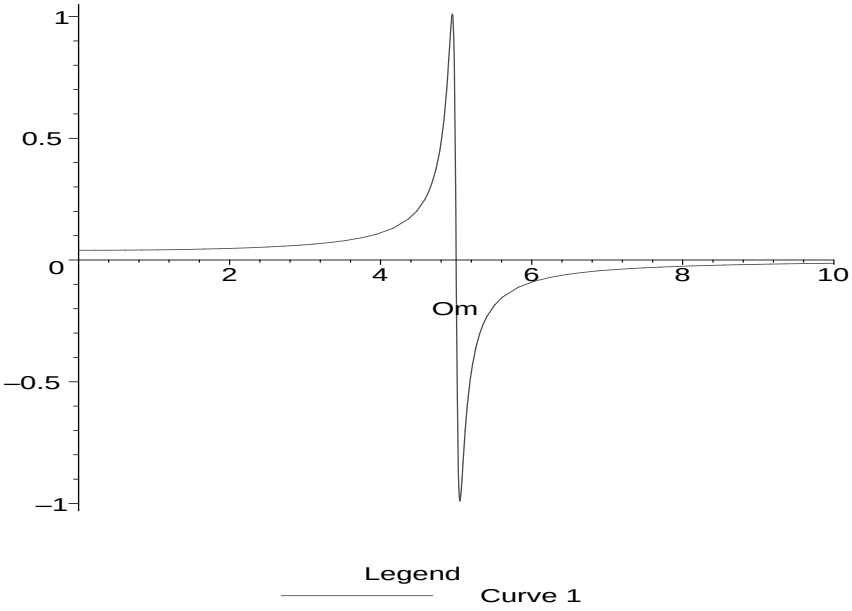
```
> A:=Om->(k-m*Om^2)/((k-m*Om^2)^2+(b*Om)^2);
```

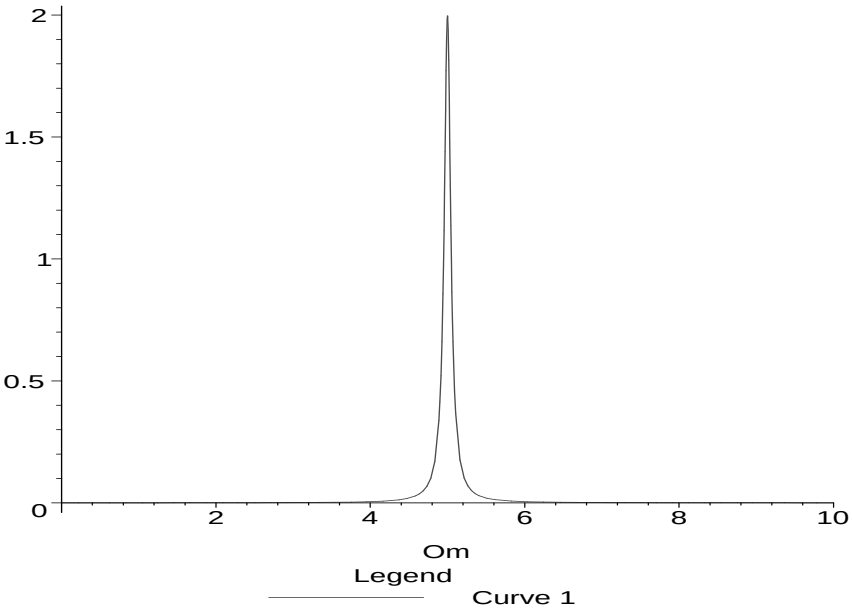
$$A := Om \rightarrow \frac{k - m\ Om^2}{(k - m\ Om^2)^2 + b^2\ Om^2}$$

```
> B:=Om->b*Om/((k-m*Om^2)^2+(b*Om)^2);
```

$$B := Om \rightarrow \frac{b\ Om}{(k - m\ Om^2)^2 + b^2\ Om^2}$$

```
> plot(A(Om), Om = 0..2*evalf(Om0));
> plot(B(Om), Om = 0..2*evalf(Om0));
```






```
> Om1=evalf(Om0);
```

$$Om1 = 5.$$