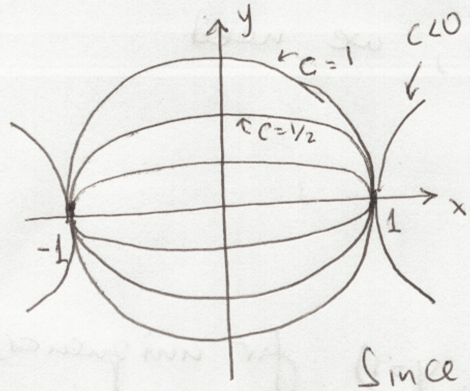
Math. 316
Set I
1.2.(16, 20, 21a, 26)

1.2.16 Verify that $x^2+cy^2=1$ (c arbitrary) is 1-param. family of implicit solutions

$$\frac{dy}{dx} = \frac{xy}{x^2-1} \quad \text{(graph.)}$$



$$\frac{d}{dx}\left(x^2+cy^2\right)=0 \Rightarrow 2x+2cy\frac{dy}{dx}=0$$

$$\Rightarrow \frac{dy}{dx} = -\frac{x}{cy} \quad (c\neq 0)$$

$$\text{Since } cy = \frac{1-x^2}{y} \Rightarrow \frac{dy}{dx} = \frac{xy}{x^2-1}$$

1.2.20 find m so $\phi(x)=e^{mx}$ satisfies:

(a) $\frac{d^2y}{dx^2}+6\frac{dy}{dx}+5y=0$? : $\frac{d^2}{dx^2}(e^{mx})+6\frac{d}{dx}(e^{mx})+5e^{mx}=$

$= (m^2+6m+5)e^{mx}=0 \Rightarrow (m+1)(m+5)=0$

$m=-1,-5$

(b) $\frac{d^3y}{dx^3}+3\frac{d^2y}{dx^2}+2\frac{dy}{dx}=0$: $\frac{d^3}{dx^3}(e^{mx})+3\frac{d^2}{dx^2}(e^{mx})+2\frac{d}{dx}(e^{mx})=$

$=(m^3+3m^2+2m)e^{mx}=0 \Rightarrow m(m+1)(m+2)=$

$\Rightarrow m=0,-1,-2$

1.2.21a find m so $y=x^m$ satisfies

$3x^2\frac{d^2y}{dx^2}+11x\frac{dy}{dx}-3y=0 \Rightarrow 3x^2\frac{d^2}{dx^2}(x^m)+11x\frac{d}{dx}(x^m)-3x^m$

$=(3m(m-1)+11m-3)x^m=0 \Rightarrow 3m^2+8m-3=0$

$m=\frac{-8\pm\sqrt{64+36}}{6}= \begin{cases} 1/3 \\ -3 \end{cases}$

$\underline{1.2.26}$ $\quad \dfrac{dy}{dx} = 3x - \sqrt[3]{y-1}\,, \quad y(2) = 1$

According to the theorem on existence/uniqueness,

~~we need~~ for $\dfrac{dy}{dx} = f(x,y)$, we need

$$f(x,y) = 3x - \sqrt[3]{y-1}$$

$$f_y(x,y) = -\frac{1}{3}(y-1)^{-2/3}$$

to be continuous at $(x=2, y=1)$ for uniqueness.

But $\displaystyle f_y(x,y)\Big|_{(2,1)} = -\frac{1}{3}(y-1)^{-2/3}\Big|_{y=1}$ is not defined

& so theorem does not apply here.