Math 316: Computer Project #1

Aircraft Guidance in a Crosswind

An aircraft flying under the guidance of a nondirectional beacon (a fixed radio transmitter, abbreviated NDB) moves so that its longitudinal axis always points toward the beacon. A pilot sets out toward an NDB from a point at which the wind is at right angles to the initial direction of the aircraft; the wind maintains this direction. Assume the windspeed and the speed of the aircraft through the air (its "airspeed") remains constant.

The variable x will represent the horizontal position of the aircraft, and y will be the vertical position. The flight will begin at the point $(2, 0)$ and end at $(0, 0)$. The differential equation describing the motion of the plane is then

$$\frac{dy}{dx} = \frac{y}{x} - \frac{W}{k}\frac{\sqrt{x^2+y^2}}{x}, \quad y(2) = 0.$$

In the above W represents the windspeed and k is the speed of the plane. From now on we will set

$$\gamma = \frac{W}{k}.$$

Begin your Maple worksheet by typing

```
> restart:
> with(DEtools): with(plots): with(plottools): with(linalg):
> assume(x,real):
> ode := diff(y(x),x)=y(x)/x-gamma*sqrt(x^2+y(x)^2)/x:
> IC := y(2)=0:
```

Part 1. We wish to see the effect of varying γ on the flight of the aircraft. In particular, we will choose $\gamma = 0.3, 0.7, 0.95, 1.05$. In order to find the explicit solution for a particular value of γ, and in order to save this solution as a function, type

```
> eq1 := subs(gamma=0.3,ode):
> soln1 := dsolve({eq1,IC}, y(x));
> y1 := unapply(rhs(soln1), x):
```

Apply this sequence of commands for each specified value of γ, so that you have created four solutions $y_1(x), \ldots, y_4(x)$. In order to plot these functions on one graph, type:

```
> plot1 := plot([y1(x),y2(x),y3(x),y4(x)], x=0..2, y=0..1,
color=[green,red,blue,black]):
> display({plot1}, title="Aircraft Flight",
titlefont = [HELVETICA, BOLD, 12],
labels = ["x","y(x)"], labelfont = [HELVETICA, BOLD, 10]);
```

a. What happens physically when $\gamma \to 1^-$?

b. What happens physically when $\gamma > 1$?

Part 2. Let us now observe how well the exact solution is approximated by a numerical solution. We will set $\gamma = 0.7$, and solve the ODE using the fourth-order Runge-Kutta method with varying stepsizes. Type

```
> plot2 := DEplot(diff(y(x),x)=y(x)/x-0.7*sqrt(x^2+y(x)^2)/x, [y(x)],
x=0.001..2, [[y(2)=0]], y=0..0.6, arrows=none, linecolor=green,
stepsize=0.5, method=classical[rk4]):
> plot3 := DEplot(diff(y(x),x)=y(x)/x-0.7*sqrt(x^2+y(x)^2)/x, [y(x)],
x=0.001..2, [[y(2)=0]], y=0..0.6, arrows=none, linecolor=brown,
stepsize=0.09, method=classical[rk4]):
> plot4 := plot([y2(x)], x=0..2, y=0..0.6, style=point,
symbol=circle, color=[red]):
> display({plot2,plot3,plot4}, title="RK4 vs. Exact",
titlefont = [HELVETICA, BOLD, 12], labels = ["x","y(x)"],
labelfont = [HELVETICA, BOLD, 10]);
```

a. Describe what you observe. In particular, explain why neither numerical approximation is doing a good job near $x = 0$.

b. In the above we used the stepsizes $h = 0.5$ and $h = 0.09$. What happens if you try $h = 0.08$? Explain why there is a problem.