Math. 316, ODEs
Practice Exam I —- Spring 2004

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1. (25pts.) Find the inverse Laplace transforms

   (a) (5pts)
   $$F(s) = \frac{1}{s(s-2)(s-3)}$$

   (b) (5pts)
   $$F(s) = \frac{1}{s^2(s^2-1)}$$

   (c) (5pts)
   $$F(s) = \frac{2s}{s^2+2s+2}$$

   (d) (5pts)
   $$F(s) = \frac{5s-4}{s^3-s^2-2s}$$

   (e) (5pts)
   $$F(s) = \frac{s^2+1}{(s^2+2s+5)^2}$$

2. (15pts.) Solve the IVP
   $$\frac{dx}{dt} = 2x - x^2 \ , \ x(0) = 1$$
   and sketch the direction field and the solution.

3. (12pts.) Give the general solution for the ODE
   $$\frac{d^2y}{dt^2} + 8\frac{dy}{dt} + k^2 y = 0$$
   when the spring constant k has the values:

   (a) $k = 5$.
   (b) $k = 4$.
   (c) $k = \sqrt{7}$.

   In each case, state if the system is overdamped, critically damped, or underdamped

4. (14pts.) Solve the IVP
   $$\frac{dy}{dt} + \frac{2t}{t^2+1}y = \frac{1}{t} \ , \ y(1) = 0$$
   and identify the largest interval in $t_1 < t < t_2$ for which the solution is defined.

5. (6pts.) Compute the Wronskian of the functions $y_1(x) = e^x$ and $y_2(x) = e^{-x}$. Are these linearly independent?

6. (14pts.) Solve the IVP using Laplace transforms:

$$\frac{d^2y}{dt^2} + y = \cos t \ , \ y(0) = y'(0) = 0 \ .$$

7. (14pts.) Solve the IVP using Laplace transforms:

$$\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = e^{-t} \ , \ y(0) = 0, y'(0) = 1 \ .$$