Project 1 Tutorial

In this tutorial I am basically working through Problem #4 on page 58 in the book Differential Equations with Maple, by Coombes et al.

```
> restart: with(plots): with(DEtools):
> de:=diff(y(x),x)=(x-exp(-x))/(y+exp(y));
```

$$de := \frac{\partial}{\partial x}\,\mathrm{y}(x) = \frac{x - \mathbf{e}^{(-x)}}{y + \mathbf{e}^{y}}$$

Let's first solve this equation the "long" way, using separation of variables.

```
> int(y+exp(y),y)-int(x-exp(-x),x);
```

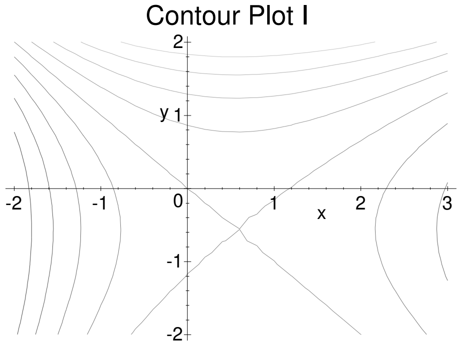
$$\frac{1}{2}y^2 + \mathbf{e}^y - \frac{1}{2}x^2 - \mathbf{e}^{(-x)}$$

```
> f:=unapply(",x,y):
> f(x,y);
```

$$\frac{1}{2}y^2 + \mathbf{e}^y - \frac{1}{2}x^2 - \mathbf{e}^{(-x)}$$

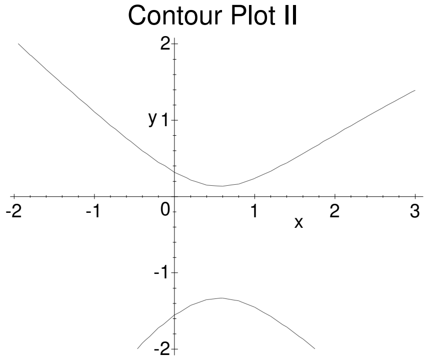
The solution is given by $\mathrm{f}(x, y) = c$, where c is determined by the initial condition. Let's see what some of the solution curves look like.

```
> contourplot(f(x,y),x=-2..3,y=-2..2, contours=10, font=[HELVETICA,BOLD,10], title=`Contour
  Plot I`,  titlefont=[HELVETICA,BOLD,14], axes=NORMAL);
```



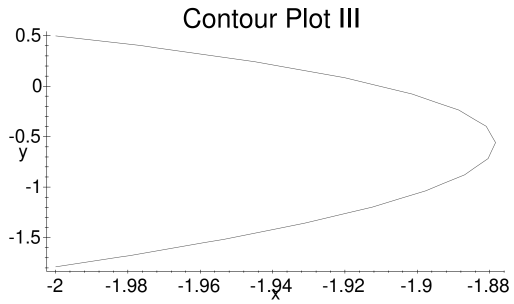
Now let's plot the solution curve corresponding to the initial condition $\mathrm{y}(1.5) = .5$.

```
> implicitplot(f(x,y)-f(1.5,0.5), x=-2..3,y=-2..2, font=[HELVETICA,BOLD,10], title=`Contour
  Plot II`,  titlefont=[HELVETICA,BOLD,14], axes=NORMAL);
```



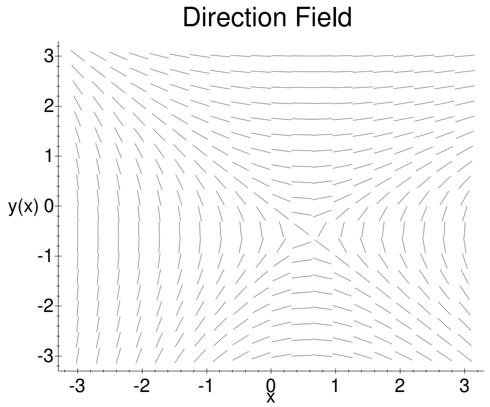
Note that there are two curves here. Given the initial condition, the upper curve is the actual solution curve. The lower curve is part of the contour plot of $\mathrm{f}(x, y) = \mathrm{f}(1.5, .5)$. Now let's look at another solution curve.

```
> implicitplot(f(x,y)-f(-2,0.5), x=-2..3,y=-2..2, font=[HELVETICA,BOLD,10], title=`Contour
  Plot III`,  titlefont=[HELVETICA,BOLD,14], axes=FRAME);
```



Now let's look at the vector field, and see what information we get.

```
> dfieldplot(de,y(x),x=-3..3,y=-3..3, font=[HELVETICA,BOLD,10], title=`Direction Field`,
  titlefont=[HELVETICA,BOLD,14], arrows=LINE, axes=FRAME);
```

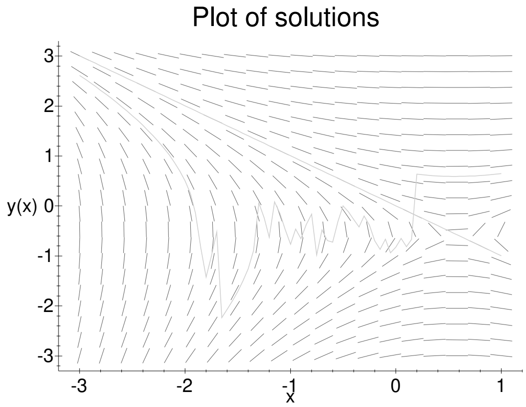


It appears that the line $y=-x$ corresponds to a solution. Check that this is actually the case.

```
>
```

Now let's add in some initial conditions, and look at the solution curves along with the vector field. It should be noted that the following command is solving the differential equation numerically.

```
> DEplot({de},[y(x)], x=-3..1,y=-3..3, [[y(-2)=2],[y(-2)=0.5]], font=[HELVETICA,BOLD,10],
  title=`Plot of solutions`, titlefont=[HELVETICA,BOLD,14], arrows=LINE, axes=FRAME,
  stepsize=0.05);
```



Note that there are problems here. Specifically, the solution with the initial condition $\mathrm{y}(-2)=.5$ doesn't look right. Why is this? (Hint: Look at Contour Plot III)

```
>
```

Now let's find the solution the "easy" way.

```
> soln:=dsolve(de,y(x));
```

$$soln := \frac{1}{2}\,\mathrm{y}(x)^2+\mathbf{e}^{\mathrm{y}(x)}-\frac{1}{2}\,x^2-\mathbf{e}^{(-x)}=\_C1$$

In order to perform all of the above calculations with this solution, we need to first let Maple know that we want to think of the solution as a function. This leads to the following command.

```
> f1:=unapply(lhs(soln),x,y);
```

$$f1 := (x, y) \to \frac{1}{2}\, y(x)^2 + \mathbf{e}^{y(x)} - \frac{1}{2}\, x^2 - \mathbf{e}^{(-x)}$$

```
> f1(x,y);
```

$$\frac{1}{2}\, \mathrm{y}(x)^2 + \mathbf{e}^{y(x)} - \frac{1}{2}\, x^2 - \mathbf{e}^{(-x)}$$

```
>
```