Project 1 Solution

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> restart: with(plots): with(DEtools):
```

Part 1

```
> de:=diff(y(x),x)=-(x^2-sin(x))/(4*y^2*cos(y));
```

$$de := \frac{\partial}{\partial x}\,y(x) = -\frac{1}{4}\,\frac{x^2 - \sin(x)}{y^2\cos(y)}$$

```
> soln:=dsolve(de,y(x));
```

$$soln := y(x)^2\sin(y(x)) - 2\sin(y(x)) + 2\,y(x)\cos(y(x)) + \frac{1}{12}x^3 + \frac{1}{4}\cos(x) = \_C1$$

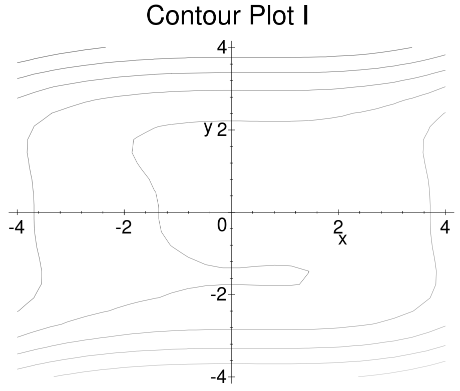
```
> f:=unapply(lhs(soln),x,y);
```

$$f := (x, y) \to y(x)^2\sin(y(x)) - 2\sin(y(x)) + 2\,y(x)\cos(y(x)) + \frac{1}{12}x^3 + \frac{1}{4}\cos(x)$$

The solution is given implicitly by $f(x, y) = f(x\_0, y\_0)$, where $y(x\_0) = y\_0$. Parts of the level curve correspond to the actual solution curve. Which parts?

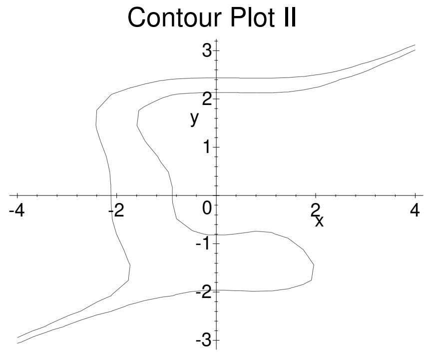
Part 2

```
> contourplot(f(x,y),x=-4..4,y=-4..4, contours=10, font=[HELVETICA,BOLD,10], title=`Contour Plot I`, titlefont=[HELVETICA,BOLD,14], axes=NORMAL);
```



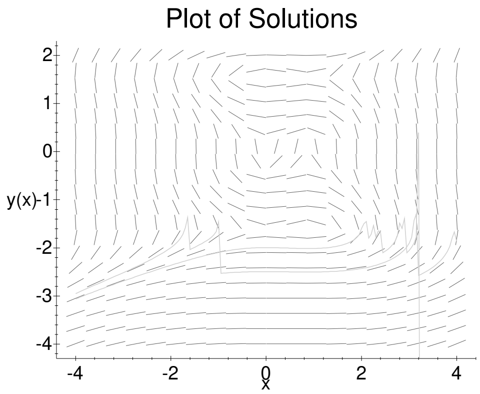
Part 3

```
> implicitplot({f(x,y)-f(-2,-2),f(x,y)-f(0,-2)},x=-4..4,y=-4..4, font=[HELVETICA,BOLD,10], title=`Contour Plot II`, titlefont=[HELVETICA,BOLD,14], axes=NORMAL);
```



Part 4

```
> DEplot({de},[y(x)] ,x=-4..4,y=-4..2, [[y(-2)=-2],[y(0)=-2]], font=[HELVETICA,BOLD,10], title=`Plot of Solutions`, titlefont=[HELVETICA,BOLD,14], arrows=LINE, axes=FRAME, stepsize=0.05);
```



The solution curves given by using "DEplot" clearly do not match those given by "implicitplot". The problem is that upon examination of the curves given by "implicitplot", it is seen that the derivative of the solution becomes infinite at some points. When the derivative is infinite, the solution no longer exists, so past that point it is foolish to expect the two curves to match. The solution with $(x\_0, y\_0) = (0, -2)$ appears to exist on the interval $[-4, 2]$, while the one with the initial condition $(x\_0, y\_0) = (-2, 2)$ appears to exist on the interval $[-4, -1.8]$.