Project 2 Tutorial

```
> restart: with(DEtools): with(plots): with(linalg):
Warning, new definition for norm
Warning, new definition for trace
```

Find the general solution to the following Euler equation.

```
> de:=t^2*diff(y(t),t$2)+t*diff(y(t),t)+y(t)=log(t)+(log(t))^2;
```

$$de := t^2 \left(\frac{\partial^2}{\partial t^2} \mathrm{y}(t)\right) + t \left(\frac{\partial}{\partial t} \mathrm{y}(t)\right) + \mathrm{y}(t) = \ln(t) + \ln(t)^2$$

Make the change of variables $x = \log t$ and write the Euler equation as a constant coefficient ODE.

```
> de_new:=(diff(z(x),x$2)-diff(z(x),x))+diff(z(x),x)+z(x)=x+x^2;
```

$$de\_new := \left(\frac{\partial^2}{\partial x^2} \mathrm{z}(x)\right) + \mathrm{z}(x) = x + x^2$$

Use the standard guess of $z = \mathbf{e}^{(r\,x)}$ to solve the homogeneous problem.

```
> de_hom:=(diff(z(x),x$2)-diff(z(x),x))+diff(z(x),x)+z(x)=0;
```

$$de\_hom := \left(\frac{\partial^2}{\partial x^2} \mathrm{z}(x)\right) + \mathrm{z}(x) = 0$$

```
> eval(subs(z(x)=exp(r*x),lhs(de_hom)));
```

$$r^2\, \mathbf{e}^{(r\,x)} + \mathbf{e}^{(r\,x)}$$

Factor out the function $\mathbf{e}^{(r\,x)}$ and find the roots of the resultant polynomial equation.

```
> simplify("/exp(r*x));
```

$$r^2 + 1$$

```
> poly:=unapply(",r);
```

$$poly := r \rightarrow r^2 + 1$$

```
> solve(poly(r),r);
```

$$I, -I$$

The solution to the homogeneous ODE is now known.

```
> z_c:=x->c1*cos(x)+c2*sin(x);
```

$$z\_c := x \rightarrow c1 \cos(x) + c2 \sin(x)$$

Now find the particular solution z_p(x) by using the method of undetermined coefficients.

```
> eval(subs(z(x)=A+B*x+C*x^2,de_new));
```

$$2\,C + A + B\,x + C\,x^2 = x + x^2$$

```
> coeff1:=A+2*C:coeff2:=B:coeff3:=C:
> solve({coeff1=0,coeff2=1,coeff3=1},{A,B,C});
```

$$\{A = \text{-}2, B = 1, C = 1\}$$

```
> z_p:=x->-2+x+x^2;
```

$$z\_p := x \rightarrow -2 + x + x^2$$

The general solution for the transformed problem is now known.

```
> z:=x->z_c(x)+z_p(x):
> z(x);
```

$$c1 \cos(x) + c2 \sin(x) - 2 + x + x^2$$

Transform back to get the general solution to the original problem.

```
> z(log(t));
```

$$c1 \cos(\ln(t)) + c2 \sin(\ln(t)) - 2 + \ln(t) + \ln(t)^2$$

```
> y_soln:=unapply(",c1,c2,t):
> y_soln(c1,c2,t);
```

$$c1 \cos(\ln(t)) + c2 \sin(\ln(t)) - 2 + \ln(t) + \ln(t)^2$$

Now that we have the general solution, use the initial conditions to determine the constants.

```
> y0:=y_soln(c1,c2,1);
```

$$y0 := c1 - 2$$

```
> diff(y_soln(c1,c2,t),t);
```

$$-\frac{c1\,\sin(\ln(t))}{t}+\frac{c2\,\cos(\ln(t))}{t}+\frac{1}{t}+2\,\frac{\ln(t)}{t}$$

```
> y_prime:=unapply(",t);
```

$$y\_prime := t \to -\frac{c1\,\sin(\ln(t))}{t}+\frac{c2\,\cos(\ln(t))}{t}+\frac{1}{t}+2\,\frac{\ln(t)}{t}$$

```
> yp0:=y_prime(1);
```

$$yp0 := 1 + c2$$

The above steps do not require that the initial condition actually be known - in the following step,
I finally write them down.

```
> solve({y0=3,yp0=-4},{c1,c2});
```

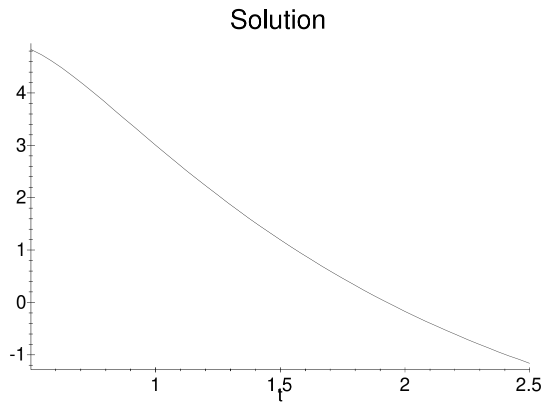
$$\{c2 = -5,\, c1 = 5\}$$

The solution to the initial value problem is now known.

```
> y_soln(5,-5,t);
```

$$5\cos(\ln(t)) - 5\sin(\ln(t)) - 2 + \ln(t) + \ln(t)^2$$

```
> plot(y_soln(5,-5,t), t=0.5..2.5, font=[HELVETICA,BOLD,10], title=`Solution`,
  titlefont=[HELVETICA,BOLD,14], axes=FRAME);
```



Now solve the ODE in one step using dsolve.

```
> ini:=y(1)=3,D(y)(1)=-4;
```

$$ini := y(1) = 3,\, D(y)(1) = -4$$

```
> dsolve({de,ini},y(t));
```

$$y(t) = 5\cos(\ln(t)) - 5\sin(\ln(t)) - 2 + \ln(t) + \ln(t)^2$$

```
>
```