Computer Project #3

Math 316
Spring 1998

Please enter your name and student ID number.

Name: _____ Student ID #: _____

Use a symbolic manipulation program such as Maple, Matlab, or Mathematica to find the exact solutions of the following problems using Laplace transforms.

1. Suppose that a *resonator* is given by the solution of the following differential equation:

$$y''(t) + 2cy'(t) + (c^2 + 4)y(t) = 10e^{-ct}\sin(2t); \quad y(0) = 0,\ y'(0) = 1, \tag{1}$$

where $c = 1/15$.

   (a) Solve this initial value problem using the following steps:
   (1) Take the Laplace transform of Eq. (1).
   (2) Solve for $Y(s)$, the Laplace transform of $y(t)$.
   (3) Take the inverse Laplace transform of $Y(s)$ to find the solution $y(t)$.

   (b) Solve the same initial value problem in one step by using a command such as dsolve (with the laplace option, if available). If your answer looks different from the one obtained by the first method, show that they are equal.

   (c) The solution in part (b) should look like

$$y(t) = f(t)\cos(2t) + g(t)\sin(2t).$$

   The amplitude function, $A(t)$, is given by

$$A(t) = \sqrt{f^2(t) + g^2(t)}.$$

   Find the maximum value of $A(t)$ and the time t at which it occurs.

   (d) Make a plot of t versus your solution $y(t)$ and include on this plot the plots of $y = A(t)$ and $y = -A(t)$. Make a second plot of $y(t)$ versus $y'(t)$. Label your plots.

2. Now suppose that the right-hand side of Eq. (1) is replaced by

$$f(t) \equiv 10u(t-20) + 5\delta(t-40)\,, \tag{2}$$

   where $u(t)$ is the step (Heaviside) function and $\delta(t)$ is the delta (impulse) function. Solve this differential equation. Make a plot of t versus your solution $y(t)$ and a plot of $y(t)$ versus $y'(t)$. Label your plots.

Hand in a print out of your complete computer algebra solutions and your four graphs. Cut all sheets down to $8\frac{1}{2}$ by 11 inches and staple in the upper left-hand corner.

Note: See the chapter on Laplace Transform Methods (Chapter 11) in *Differential Equations with Maple, 2nd Edition*, by K. Coombes *et. al.*, for help with this project.