Project 3 Tutorial

Solve an ODE using the Laplace transform..

```
> restart: with(DEtools): with(inttrans): with(plots):
> de:=diff(y(t),t$2)+9*y(t)=3*Heaviside(t-2)+4*Dirac(t-4);
```

$$de := \left(\frac{\partial^2}{\partial t^2}\mathrm{y}(t)\right) + 9\,\mathrm{y}(t) = 3\,\mathrm{Heaviside}(t-2) + 4\,\mathrm{Dirac}(t-4)$$

```
> ini:=y(0)=0,D(y)(0)=0:
```

Take the Laplace transform of the above ODE, and then solve for the Laplace transform of the solution. This is the long way of finding the solution.

```
> Leq:=laplace(de,t,s):
> LY:=solve(Leq,laplace(y(t),t,s));
```

$$LY := \frac{s^2\,\mathrm{y}(0) + \mathrm{D}(y)(0)\,s + 3\,\mathbf{e}^{(-2\,s)} + 4\,\mathbf{e}^{(-4\,s)}\,s}{s\,(s^2+9)}$$

```
> LY_part:=subs(ini,LY);
```

$$LY\_part := \frac{3\,\mathbf{e}^{(-2\,s)} + 4\,\mathbf{e}^{(-4\,s)}\,s}{s\,(s^2+9)}$$

```
> sol:=invlaplace(LY_part,s,t);
```

$$sol := \frac{1}{3}\mathrm{Heaviside}(t-2) - \frac{1}{3}\mathrm{Heaviside}(t-2)\cos(3\,t-6) + \frac{4}{3}\mathrm{Heaviside}(t-4)\sin(3\,t-12)$$

Now, instead of repeating the above steps, use the built-in solver supplied by Maple and solve the ODE in one step.

```
> dsolve({de,ini},y(t),method=laplace);
```

$$\mathrm{y}(t) = \frac{1}{3}\mathrm{Heaviside}(t-2) - \frac{1}{3}\mathrm{Heaviside}(t-2)\cos(3\,t-6) + \frac{4}{3}\mathrm{Heaviside}(t-4)\sin(3\,t-12)$$

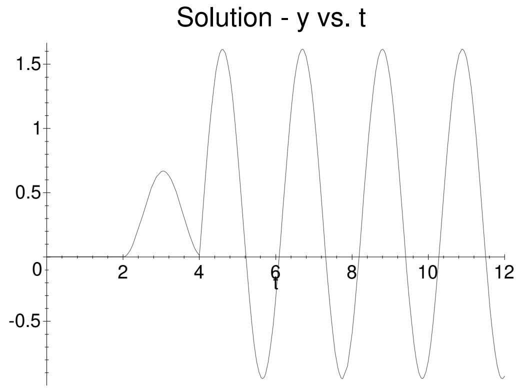
At the moment, the solution is not really a function of *t*. The following statement will make the solution a function.

```
> y_sol:=unapply(rhs("),t);
```

$$y\_sol := t \rightarrow \frac{1}{3}\mathrm{Heaviside}(t-2) - \frac{1}{3}\mathrm{Heaviside}(t-2)\cos(3\,t-6) + \frac{4}{3}\mathrm{Heaviside}(t-4)\sin(3\,t-12)$$

Plot the solution for *t* between 0 and 12.

```
> plot(y_sol(t),t=0..12, font=[HELVETICA,BOLD,10], title=`Solution - y vs. t`,
  titlefont=[HELVETICA,BOLD,14], axes=NORMAL);
```



Suppose that you wish to find the zero of the solution y(t) on the interval [8,9].

```
> tzero:=fsolve(y_sol(t),t,8..9);
```

$$tzero := 8.185216783$$

Check to see that it's correct. As you can see, it's a pretty good approximation.

```
> y_sol(tzero);
```

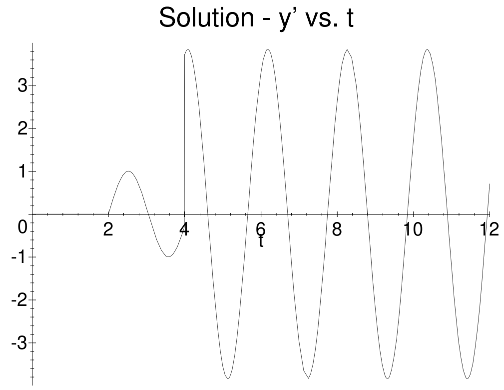
$$.28\ 10^{-9}$$

Finally, plot the solution $y(t)$ versus $\frac{\partial}{\partial t} y(t)$.

```
> diff(y_sol(t),t);
```

$$\frac{1}{3}\,\mathrm{Dirac}(t-2)-\frac{1}{3}\,\mathrm{Dirac}(t-2)\cos(3\,t-6)+\mathrm{Heaviside}(t-2)\sin(3\,t-6)+\frac{4}{3}\,\mathrm{Dirac}(t-4)\sin(3\,t-12)$$
$$+\,4\,\mathrm{Heaviside}(t-4)\cos(3\,t-12)$$

```
> dy_sol:=unapply(",t):
> plot(dy_sol(t),t=0..12, font=[HELVETICA,BOLD,10], title=`Solution - y' vs. t`,
  titlefont=[HELVETICA,BOLD,14], axes=NORMAL);
```




```
> plot([y_sol(t),dy_sol(t),t=0..12], font=[HELVETICA,BOLD,10], title=`Solution - y' vs. y`,
  titlefont=[HELVETICA,BOLD,14], axes=NORMAL);
```