# Project 3 Solution

## Part 1

```
> restart: with(inttrans): with(DEtools): with(plots):
> de:=diff(y(t),t$2)+2/15*diff(y(t),t)+((1/15)^2+2^2)*y(t)=10*exp(-1/15*t)*sin(2*t);
```

$$de := \left(\frac{\partial^2}{\partial t^2}\,\mathrm{y}(t)\right) + \frac{2}{15}\left(\frac{\partial}{\partial t}\,\mathrm{y}(t)\right) + \frac{901}{225}\,\mathrm{y}(t) = 10\,\mathbf{e}^{(-1/15\,t)}\sin(2\,t)$$

```
> ini:=y(0)=0,D(y)(0)=1:
```

Take the Laplace transform of the above ODE, and then solve for the Laplace transform of the solution.

```
> Leq:=laplace(de,t,s):
> LY:=solve(Leq,laplace(y(t),t,s));
```

$$LY := 15\,\frac{3375\,\mathrm{y}(0)\,s^3 + 900\,\mathrm{y}(0)\,s^2 + 13575\,\mathrm{y}(0)\,s + 3375\,\mathrm{D}(y)(0)\,s^2 + 450\,\mathrm{D}(y)(0)\,s + 13515\,\mathrm{D}(y)(0) + 1802\,\mathrm{y}(0) + 67500}{50625\,s^4 + 13500\,s^3 + 406350\,s^2 + 54060\,s + 811801}$$

```
> LY_part:=subs({y(0)=0,D(y)(0)=1},LY);
```

$$LY\_part := 15\,\frac{81015 + 3375\,s^2 + 450\,s}{50625\,s^4 + 13500\,s^3 + 406350\,s^2 + 54060\,s + 811801}$$

```
> sol:=invlaplace(LY_part,s,t);
```

$$sol := -\frac{5}{2}\,\mathbf{e}^{(-1/15\,t)}\,t\cos(2\,t) + \frac{7}{4}\,\mathbf{e}^{(-1/15\,t)}\sin(2\,t)$$

Now, instead of repeating the above steps, use the built-in solver supplied by Maple and solve the ODE in one step.

```
> dsolve({de,ini},y(t),method=laplace);
```

$$\mathrm{y}(t) = -\frac{5}{2}\,\mathbf{e}^{(-1/15\,t)}\,t\cos(2\,t) + \frac{7}{4}\,\mathbf{e}^{(-1/15\,t)}\sin(2\,t)$$

```
> y_sol:=unapply(rhs("),t);
```

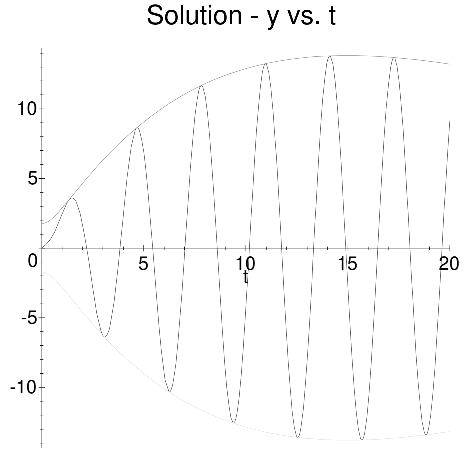
$$y\_sol := t \rightarrow -\frac{5}{2}\,\mathbf{e}^{(-1/15\,t)}\,t\cos(2\,t) + \frac{7}{4}\,\mathbf{e}^{(-1/15\,t)}\sin(2\,t)$$

Define the amplitude function A($t$) using the given formula given in class.

```
> A:=t->sqrt((-5/2*t*exp(-1/15*t))^2+(7/4*exp(-1/15*t))^2);
```

$$A := t \rightarrow \sqrt{\frac{25}{4}\,t^2\left(\mathbf{e}^{(-1/15\,t)}\right)^2 + \frac{49}{16}\left(\mathbf{e}^{(-1/15\,t)}\right)^2}$$

```
> plot([y_sol(t),A(t),-A(t)], t=0..20, font=[HELVETICA,BOLD,10], title='Solution - y vs. t', titlefont=[HELVETICA,BOLD,14], axes=NORMAL);
```




From the above graph it is clear that A($t$) has a maximum. We will now find that maximum and the point at which it occurs. It should be obvious as to why I am using the derivative of A($t$).

```
> diff(A(t),t);
```

$$\frac{1}{8}\,\frac{200\,t\,(\mathbf{e}^{(-1/15\,t)})^2-\frac{40}{3}\,t^2\,(\mathbf{e}^{(-1/15\,t)})^2-\frac{98}{15}\,(\mathbf{e}^{(-1/15\,t)})^2}{\sqrt{100\,t^2\,(\mathbf{e}^{(-1/15\,t)})^2+49\,(\mathbf{e}^{(-1/15\,t)})^2}}$$

```
> Aprime:=unapply(",t):
> t_max:=fsolve(Aprime(t),t,10..40);
```

$$t\_max := 14.96726188$$

```
> A_max:=A(t_max);
```

$$A\_max := 13.81052540$$

# Part 2

Now we will solve the ODE with a different right-hand side which involves the Heaviside function and the delta function. After defining the ODE, I will solve it in one step.

```
> de1:=diff(y(t),t$2)+2/15*diff(y(t),t)+((1/15)^2+2^2)*y(t)=10*Heaviside(t-10*2)+5*Dirac(t-
  20*2);
```

$$de1 := \left(\frac{\partial^2}{\partial t^2}\,\mathrm{y}(t)\right)+\frac{2}{15}\left(\frac{\partial}{\partial t}\,\mathrm{y}(t)\right)+\frac{901}{225}\,\mathrm{y}(t)=10\ \mathrm{Heaviside}(t-20)+5\ \mathrm{Dirac}(t-40)$$

```
> dsolve({de1,ini},y(t),method=laplace);
```

$$\mathrm{y}(t)=\frac{1}{2}\,\mathbf{e}^{(-1/15\,t)}\sin(2\,t)+\frac{2250}{901}\,\mathrm{Heaviside}(t-20)-\frac{2250}{901}\,\mathrm{Heaviside}(t-20)\,\mathbf{e}^{(-1/15\,t+4/3)}\cos(2\,t-40)$$
$$-\frac{75}{901}\,\mathrm{Heaviside}(t-20)\,\mathbf{e}^{(-1/15\,t+4/3)}\sin(2\,t-40)+\frac{5}{2}\,\mathrm{Heaviside}(t-40)\,\mathbf{e}^{(-1/15\,t+8/3)}\sin(2\,t-80)$$

```
> Y:=unapply(rhs("),t):
```

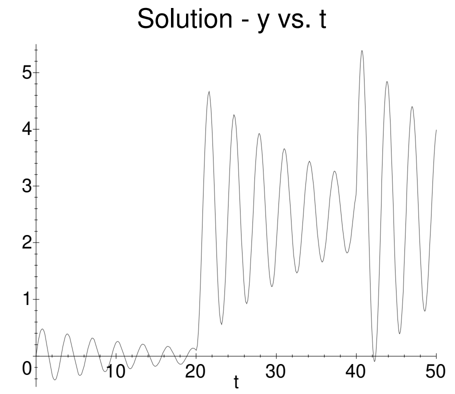
Since we will want to graph the solution y($t$) versus its derivative $\frac{\partial}{\partial t}\,\mathrm{y}(t)$, we will need to find the derivative of y($t$). Recall that the derivative of the Heaviside function is the delta function.

```
> diff(Y(t),t):
> Yprime:=unapply(",t);
```

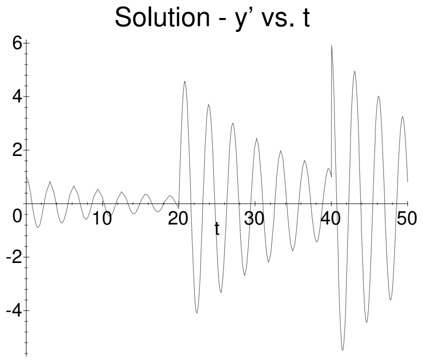
$$Yprime := t \to -\frac{1}{30}\,\mathbf{e}^{(-1/15\,t)}\sin(2\,t)+\mathbf{e}^{(-1/15\,t)}\cos(2\,t)+\frac{2250}{901}\,\mathrm{Dirac}(t-20)-\frac{2250}{901}\,\mathrm{Dirac}(t-20)\,\mathbf{e}^{(-1/15\,t+4/3)}\cos(2\,t-40)$$
$$+5\ \mathrm{Heaviside}(t-20)\,\mathbf{e}^{(-1/15\,t+4/3)}\sin(2\,t-40)-\frac{75}{901}\,\mathrm{Dirac}(t-20)\,\mathbf{e}^{(-1/15\,t+4/3)}\sin(2\,t-40)$$
$$+\frac{5}{2}\,\mathrm{Dirac}(t-40)\,\mathbf{e}^{(-1/15\,t+8/3)}\sin(2\,t-80)-\frac{1}{6}\,\mathrm{Heaviside}(t-40)\,\mathbf{e}^{(-1/15\,t+8/3)}\sin(2\,t-80)$$
$$+5\ \mathrm{Heaviside}(t-40)\,\mathbf{e}^{(-1/15\,t+8/3)}\cos(2\,t-80)$$

Although it's not necessary to do both plots, I am including a plot of y($t$) and $\frac{\partial}{\partial t}\,\mathrm{y}(t)$ below, just so that we have an idea as to what these functions look like. These plots may also help in interpreting the final plot below.

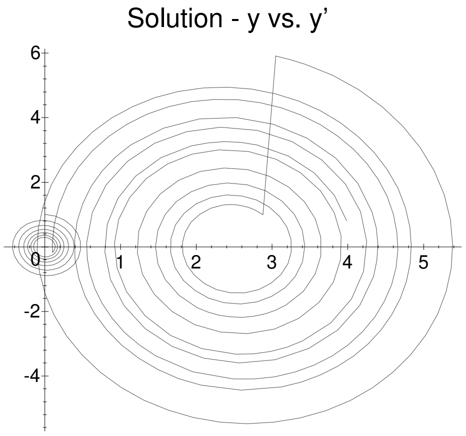
```
> plot(Y(t), t=0..50, font=[HELVETICA,BOLD,10], title=`Solution - y vs. t`,
  titlefont=[HELVETICA,BOLD,14], axes=NORMAL);
```




```
> plot(Yprime(t), t=0..50, font=[HELVETICA,BOLD,10], title=`Solution - y' vs. t`,
  titlefont=[HELVETICA,BOLD,14], axes=NORMAL);
```




```
> plot([Y(t),Yprime(t),t=0..50], font=[HELVETICA,BOLD,10], title=`Solution - y vs. y'`,
  titlefont=[HELVETICA,BOLD,14], axes=NORMAL);
```




```
>
```