COMPUTER PROJECT #4

MATH 316

Spring 1998

Consider the three linear systems of ODEs:

1. $$\mathbf{x}' = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix} \mathbf{x} + \begin{bmatrix} e^t \\ t \end{bmatrix}, \qquad \mathbf{x}(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

2. $$\mathbf{x}' = \begin{bmatrix} 2 & -5 \\ 1 & -2 \end{bmatrix} \mathbf{x} + \begin{bmatrix} -\cos t \\ \sin t \end{bmatrix}, \qquad \mathbf{x}(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

3. $$\mathbf{x}' = \begin{bmatrix} -3 & \sqrt{2} \\ \sqrt{2} & -2 \end{bmatrix} \mathbf{x} + \begin{bmatrix} e^{-t} \\ e^{-t} \end{bmatrix}, \qquad \mathbf{x}(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

For the first two ODEs, do the following:

1. Find the eigenvalues and eigenvectors of the matrix, and use these to write down the general solution to the homogeneous problem.

2. Find the fundamental matrix solution $\Phi(t)$.

3. Find a particular solution $\mathbf{x}_p(t)$ of the nonhomogeneous ODE, and then find the solution $\mathbf{x}(t)$. Plot the two components of the vector solution $\mathbf{x}(t)$ on the same graph for $-1 \leq t \leq 1$, and then on another graph plot $x_2(t)$ versus $x_1(t)$. For the second graph, use the differential equation to indicate the direction of motion through the initial value.

4. Expand the time interval and, purely on the graphical evidence, venture a guess as to the behavior of the solution curve as $t \to -\infty$ and $t \to +\infty$.

For the last ODE, use **dsolve** to solve the ODE, plot $x_2(t)$ versus $x_1(t)$ for $-1 \leq t \leq 1$ and indicate the direction of motion through the initial value, and then venture a guess as to the behavior of the solution curve as $t \to -\infty$ and $t \to +\infty$.

Hand in a print out of your complete computer algebra solutions and your graphs. Cut all sheets down to $8\frac{1}{2}$ by 11 inches and staple in the upper left-hand corner.