Project 4 Tutorial

```
> restart: with(linalg): with(DEtools): with(inttrans): with(plots):
Warning, new definition for norm
Warning, new definition for trace
Warning, new definition for hilbert
```

Find the linearly independent solutions for the following matrix.

```
> A:=matrix(2,2,[[-3,-1],[2,-1]]);
```

$$A := \begin{bmatrix} -3 & -1 \\ 2 & -1 \end{bmatrix}$$

```
> eigenvects(A);
```

$$[-2+I, 1, \{[1, -1-I]\}], [-2-I, 1, \{[1, -1+I]\}]$$

Get the real and imaginary parts of the eigenvector corresponding to the eigenvalue $-2+i$.

```
> p:=vector([1,-1]);
```

$$p := [1, -1]$$

```
> q:=vector([0,-1]);
```

$$q := [0, -1]$$

```
> y_1:=t->evalm(exp(-2*t)*(cos(t)*p-sin(t)*q)): y_1(t);
```

$$[\mathbf{e}^{(-2t)}\cos(t), \mathbf{e}^{(-2t)}(-\cos(t)+\sin(t))]$$

```
> y_2:=t->evalm(exp(-2*t)*(sin(t)*p+cos(t)*q)): y_2(t);
```

$$[\mathbf{e}^{(-2t)}\sin(t), \mathbf{e}^{(-2t)}(-\sin(t)-\cos(t))]$$

The general solution is then given by

```
> y_h:=t->evalm(c_1*y_1(t)+c_2*y_2(t)): y_h(t);
```

$$[c_1\,\mathbf{e}^{(-2t)}\cos(t)+c_2\,\mathbf{e}^{(-2t)}\sin(t), c_1\,\mathbf{e}^{(-2t)}(-\cos(t)+\sin(t))+c_2\,\mathbf{e}^{(-2t)}(-\sin(t)-\cos(t))]$$

Use the Laplace transform to find the fundamental matrix solution $\Phi(t)$.

```
> I_2:=array(1..2,1..2,identity):
> Phihat:=s->evalm(inverse(s*I_2-A)): Phihat(s);
```

$$\begin{bmatrix} \dfrac{s+1}{s^2+4s+5} & -\dfrac{1}{s^2+4s+5} \\ \dfrac{2}{s^2+4s+5} & \dfrac{s+3}{s^2+4s+5} \end{bmatrix}$$

```
> Phi:=t->map(invlaplace,Phihat(s),s,t): Phi(t);
```

$$\begin{bmatrix} \mathbf{e}^{(-2t)}\cos(t)-\%1 & -\%1 \\ 2\,\%1 & \mathbf{e}^{(-2t)}\cos(t)+\%1 \end{bmatrix}$$

$$\%1 := \mathbf{e}^{(-2t)}\sin(t)$$

For the following vector function, let us now find the particular solution (again using the Laplace transform).

```
> f:=t->vector([exp(3*t),exp(2*t)]): f(t);
```

$$[\mathbf{e}^{(3t)}, \mathbf{e}^{(2t)}]$$

```
> F:=s->map(laplace,f(t),t,s): F(s);
```

$$\left[\frac{1}{s-3}, \frac{1}{s-2}\right]$$

```
> Yp:=s->evalm(Phihat(s)&*F(s)): Yp(s);
```

$$\left[\frac{s+1}{(s^2+4s+5)(s-3)}-\frac{1}{(s^2+4s+5)(s-2)}, \frac{2}{(s^2+4s+5)(s-3)}+\frac{s+3}{(s^2+4s+5)(s-2)}\right]$$

```
> y_p:=t->map(invlaplace,Yp(s),s,t): y_p(t);
```

$$\left[\frac{2}{13}\mathbf{e}^{(3t)}-\frac{21}{221}\mathbf{e}^{(-2t)}\cos(t)+\frac{103}{221}\mathbf{e}^{(-2t)}\sin(t)-\frac{1}{17}\mathbf{e}^{(2t)}, \frac{1}{13}\mathbf{e}^{(3t)}-\frac{82}{221}\mathbf{e}^{(-2t)}\cos(t)-\frac{124}{221}\mathbf{e}^{(-2t)}\sin(t)+\frac{5}{17}\mathbf{e}^{(2t)}\right]$$

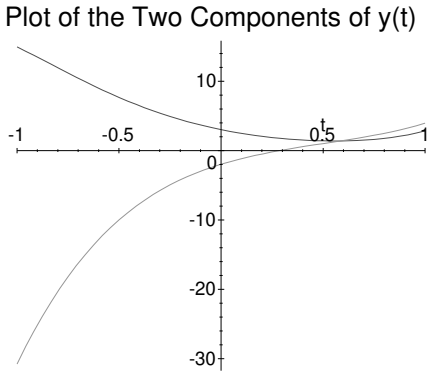
Now let us write down the solution for the full ODE.

```
> y_0:=vector([3,-2]):
> y:=t->evalm(Phi(t)&*y_0 + y_p(t)): y(t);
```

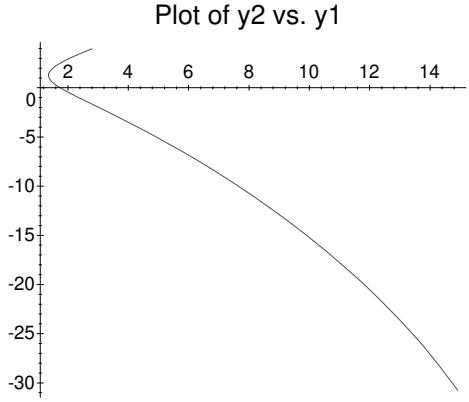
$$\left[\frac{642}{221}\mathbf{e}^{(-2t)}\cos(t)-\frac{118}{221}\mathbf{e}^{(-2t)}\sin(t)+\frac{2}{13}\mathbf{e}^{(3t)}-\frac{1}{17}\mathbf{e}^{(2t)}, \frac{760}{221}\mathbf{e}^{(-2t)}\sin(t)-\frac{524}{221}\mathbf{e}^{(-2t)}\cos(t)+\frac{1}{13}\mathbf{e}^{(3t)}+\frac{5}{17}\mathbf{e}^{(2t)}\right]$$

Now plot the different solution components.

```
> plot({y(t)[1],y(t)[2]}, t=-1..1, font=[HELVETICA,BOLD,10], title=`Plot of the Two
  Components of y(t)`, titlefont=[HELVETICA,BOLD,14], axes=NORMAL);
```




```
> plot([y(t)[1],y(t)[2], t=-1..1], font=[HELVETICA,BOLD,10], title=`Plot of y2 vs. y1`,
  titlefont=[HELVETICA,BOLD,14], axes=NORMAL);
```



Use the command "dsolve" to solve the above ODE.

```
> f_1:=(w,z)->-3*w-z+exp(3*t): f_1(w,z);
```

$$-3\,w - z + \mathbf{e}^{(3\,t)}$$

```
> f_2:=(w,z)->2*w-z+exp(2*t): f_2(w,z);
```

$$2\,w - z + \mathbf{e}^{(2\,t)}$$

```
> dsolve({diff(w(t),t)=f_1(w(t),z(t)),diff(z(t),t)=f_2(w(t),z(t)),w(0)=3,z(0)=-2},
  {w(t),z(t)}, method=laplace);
```

$$\{\mathrm{w}(t) = \frac{642}{221}\,\mathbf{e}^{(-2\,t)}\cos(t) - \frac{118}{221}\,\mathbf{e}^{(-2\,t)}\sin(t) - \frac{1}{17}\,\mathbf{e}^{(2\,t)} + \frac{2}{13}\,\mathbf{e}^{(3\,t)},\ \mathrm{z}(t) = \frac{760}{221}\,\mathbf{e}^{(-2\,t)}\sin(t) - \frac{524}{221}\,\mathbf{e}^{(-2\,t)}\cos(t) + \frac{5}{17}\,\mathbf{e}^{(2\,t)} + \frac{1}{13}\,\mathbf{e}^{(3\,t)}\}$$

```
> soln_temp:=":
> w_temp:=soln_temp[1]: w:=unapply(rhs(w_temp),t): w(t);
```

$$\frac{642}{221}\,\mathbf{e}^{(-2\,t)}\cos(t) - \frac{118}{221}\,\mathbf{e}^{(-2\,t)}\sin(t) - \frac{1}{17}\,\mathbf{e}^{(2\,t)} + \frac{2}{13}\,\mathbf{e}^{(3\,t)}$$

```
> z_temp:=soln_temp[2]: z:=unapply(rhs(z_temp),t): z(t);
```

$$\frac{760}{221}\,\mathbf{e}^{(-2\,t)}\sin(t) - \frac{524}{221}\,\mathbf{e}^{(-2\,t)}\cos(t) + \frac{5}{17}\,\mathbf{e}^{(2\,t)} + \frac{1}{13}\,\mathbf{e}^{(3\,t)}$$

We can now do all the plotting with the solutions w(t) and z(t).....