Project 4 Solution

```
> restart: with(linalg): with(DEtools): with(inttrans): with(plots):
Warning, new definition for norm
Warning, new definition for trace
Warning, new definition for hilbert
```

Problem 1

Part 1

Find the linearly independent solutions for the following matrix.

```
> A1:=matrix([[2,-1],[3,-2]]);
```

$$A1 := \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}$$

```
> eigenvects(A1);
```

$$[1, 1, \{[1, 1]\}], [-1, 1, \{[1, 3]\}]$$

```
> v1_1:=vector([1,1]):   v1_2:=vector([1,3]):
> x1_1:=t->evalm(exp(t)*v1_1):   x1_2:=t->evalm(exp(-t)*v1_2):
```

The general solution to the homogeneous problem is then given by

```
> x1_h:=t->evalm(c1*x1_1(t)+c2*x1_2(t)):   x1_h(t);
```

$$[c1\,\mathbf{e}^t + c2\,\mathbf{e}^{(-t)}, c1\,\mathbf{e}^t + 3\,c2\,\mathbf{e}^{(-t)}]$$

Part 2

Use the Laplace transform to find the fundamental matrix solution $\Phi(t)$.

```
> I_2:=array(1..2,1..2,identity):
> Phihat1:=s->evalm(inverse(s*I_2-A1)):
> Phi1:=t->map(invlaplace,Phihat1(s),s,t): Phi1(t);
```

$$\begin{bmatrix} \frac{3}{2}\mathbf{e}^t - \frac{1}{2}\mathbf{e}^{(-t)} & -\frac{1}{2}\mathbf{e}^t + \frac{1}{2}\mathbf{e}^{(-t)} \\ \frac{3}{2}\mathbf{e}^t - \frac{3}{2}\mathbf{e}^{(-t)} & -\frac{1}{2}\mathbf{e}^t + \frac{3}{2}\mathbf{e}^{(-t)} \end{bmatrix}$$

Part 3

Now find the particular solution (again using the Laplace transform).

```
> f1:=t->vector([exp(t),t]):
> F1:=s->map(laplace,f1(t),t,s):
> X1p:=s->evalm(Phihat1(s)&*F1(s)):
> x1_p:=t->map(invlaplace,X1p(s),s,t): x1_p(t);
```

$$\left[\frac{3}{2}\,t\,\mathbf{e}^t - \frac{3}{4}\mathbf{e}^t + \frac{3}{4}\mathbf{e}^{(-t)} + t, \frac{3}{2}\,t\,\mathbf{e}^t - \frac{5}{4}\mathbf{e}^t + \frac{9}{4}\mathbf{e}^{(-t)} + 2\,t - 1\right]$$

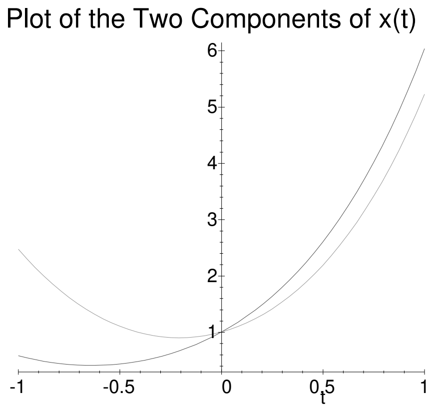
Now let us write down the solution for the full ODE.

```
> x_0:=vector([1,1]):
> x1:=t->evalm(Phi1(t)&*x_0 + x1_p(t)): x1(t);
```

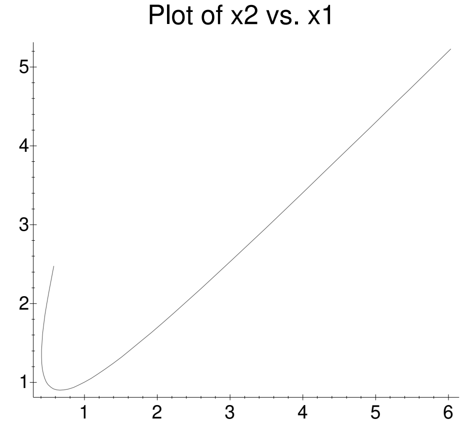
$$\left[\frac{1}{4}\mathbf{e}^t + \frac{3}{2}\,t\,\mathbf{e}^t + \frac{3}{4}\mathbf{e}^{(-t)} + t, -\frac{1}{4}\mathbf{e}^t + \frac{3}{2}\,t\,\mathbf{e}^t + \frac{9}{4}\mathbf{e}^{(-t)} + 2\,t - 1\right]$$

Now plot the different solution components.

```
> plot({x1(t)[1],x1(t)[2]}, t=-1..1, font=[HELVETICA,BOLD,10], title=`Plot of the Two
  Components of x(t)`, titlefont=[HELVETICA,BOLD,14], axes=NORMAL);
```



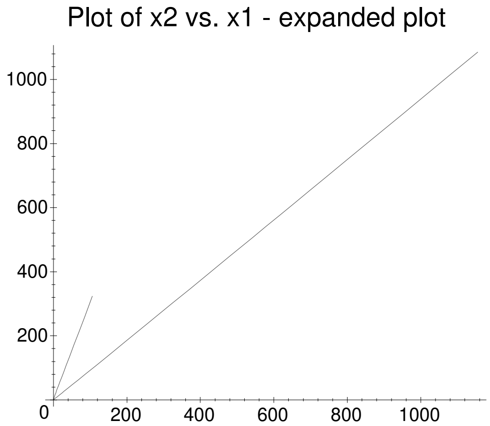

```
> plot([x1(t)[1],x1(t)[2], t=-1..1], font=[HELVETICA,BOLD,10], title=`Plot of x2 vs. x1`,
  titlefont=[HELVETICA,BOLD,14], axes=NORMAL);
```



Part 4

Expand the time interval for the above plot, and use the graphical evidence to conjecture what happens as t approaches plus and minus ∞ .

```
> plot([x1(t)[1],x1(t)[2], t=-5..5], font=[HELVETICA,BOLD,10], title=`Plot of x2 vs. x1 -
  expanded plot`, titlefont=[HELVETICA,BOLD,14], axes=NORMAL);
```



It appears that both components of the solution will approach infinity as t approaches plus and minus ∞.

Problem 2

Part 1

Find the linearly independent solutions for the following matrix.

```
> A2:=matrix([[2,-5],[1,-2]]);
```

$$A2 := \begin{bmatrix} 2 & -5 \\ 1 & -2 \end{bmatrix}$$

```
> eigenvects(A2);
```

$$[I, 1, \{[2+I, 1]\}], [-I, 1, \{[2-I, 1]\}]$$

```
> p2:=vector([2,1]):   q2:=vector([1,0]):
> x2_1:=t->evalm(cos(t)*p2-sin(t)*q2):   x2_2:=t->evalm(sin(t)*p2+cos(t)*q2):
```

The general solution to the homogeneous problem is then given by

```
> x2_h:=t->evalm(c1*x2_1(t)+c2*x2_2(t)):   x2_h(t);
```

$$[c1\,(2\cos(t)-\sin(t))+c2\,(2\sin(t)+\cos(t)),\ c1\cos(t)+c2\sin(t)]$$

Part 2

Use the Laplace transform to find the fundamental matrix solution $\Phi(t)$.

```
> I_2:=array(1..2,1..2,identity):
> Phihat2:=s->evalm(inverse(s*I_2-A2)):
> Phi2:=t->map(invlaplace,Phihat2(s),s,t): Phi2(t);
```

$$\begin{bmatrix} \cos(t)+2\sin(t) & -5\sin(t) \\ \sin(t) & \cos(t)-2\sin(t) \end{bmatrix}$$

Part 3

Now find the particular solution (again using the Laplace transform).

```
> f2:=t->vector([-cos(t),sin(t)]):
> F2:=s->map(laplace,f2(t),t,s):
> X2p:=s->evalm(Phihat2(s)&*F2(s)):
> x2_p:=t->map(invlaplace,X2p(s),s,t): x2_p(t);
```

$$[2\,t\cos(t)-t\sin(t)-3\sin(t),\ t\cos(t)-\sin(t)]$$

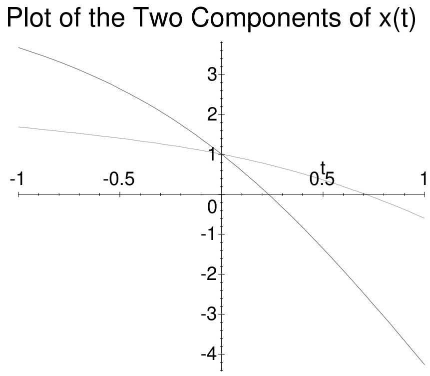
Now let us write down the solution for the full ODE.

```
> x_0:=vector([1,1]):
> x2:=t->evalm(Phi2(t)&*x_0 + x2_p(t)): x2(t);
```

$$[\cos(t)-6\sin(t)+2\,t\cos(t)-t\sin(t),\ -2\sin(t)+\cos(t)+t\cos(t)]$$

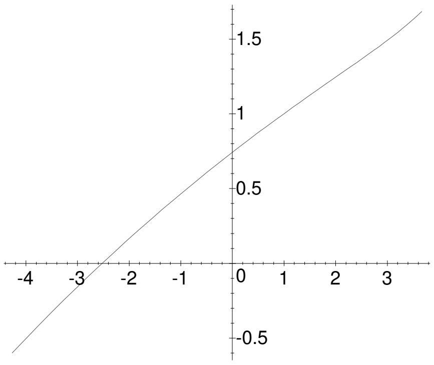
Now plot the different solution components.

```
> plot({x2(t)[1],x2(t)[2]}, t=-1..1, font=[HELVETICA,BOLD,10], title=`Plot of the Two
  Components of x(t)`, titlefont=[HELVETICA,BOLD,14], axes=NORMAL);
```




```
> plot([x2(t)[1],x2(t)[2], t=-1..1], font=[HELVETICA,BOLD,10], title=`Plot of x2 vs. x1`,
  titlefont=[HELVETICA,BOLD,14], axes=NORMAL);
```

Plot of x2 vs. x1

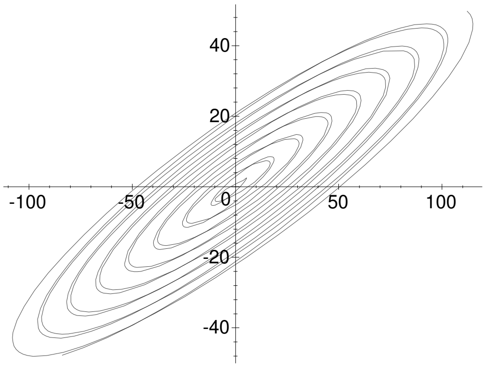


Part 4

Expand the time interval for the above plot, and use the graphical evidence to conjecture what happens as *t* approaches plus and minus ∞ .

```
> plot([x2(t)[1],x2(t)[2], t=-50..50], font=[HELVETICA,BOLD,10], title=`Plot of x2 vs. x1 -
  expanded plot`, titlefont=[HELVETICA,BOLD,14], axes=NORMAL);
```

Plot of x2 vs. x1 - expanded plot



It appears that both components of the solution will approach infinity as *t* approaches plus and minus ∞. This can be verified by looking at the actual solution.

Problem 3

Use the command "dsolve" to solve the above ODE.

```
> g1:=(x,y)->-3*x+sqrt(2)*y+exp(-t):
> g2:=(x,y)->sqrt(2)*x-2*y+exp(-t):
> dsolve({diff(x(t),t)=g1(x(t),y(t)),diff(y(t),t)=g2(x(t),y(t)),x(0)=1,y(0)=1},
  {x(t),y(t)}, method=laplace);
```

$$\{\,x(t)=\frac{4}{9}\mathbf{e}^{(-4\,t)}-\frac{2}{9}\mathbf{e}^{(-4\,t)}\sqrt{2}+\frac{5}{9}\mathbf{e}^{(-t)}+\frac{2}{9}\mathbf{e}^{(-t)}\sqrt{2}+\frac{1}{3}t\,\mathbf{e}^{(-t)}+\frac{1}{3}t\,\mathbf{e}^{(-t)}\sqrt{2},$$

$$y(t)=\frac{2}{9}\mathbf{e}^{(-4\,t)}-\frac{2}{9}\mathbf{e}^{(-4\,t)}\sqrt{2}+\frac{7}{9}\mathbf{e}^{(-t)}+\frac{2}{9}\mathbf{e}^{(-t)}\sqrt{2}+\frac{2}{3}t\,\mathbf{e}^{(-t)}+\frac{1}{3}t\,\mathbf{e}^{(-t)}\sqrt{2}\,\}$$

```
> soln_temp:=":
> x_temp:=soln_temp[1]: x:=unapply(rhs(x_temp),t): x(t);
```

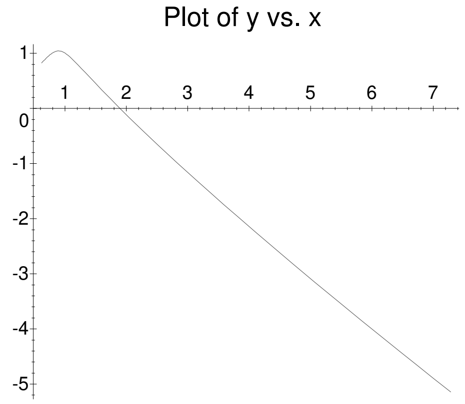
$$\frac{4}{9}\mathbf{e}^{(-4\,t)}-\frac{2}{9}\mathbf{e}^{(-4\,t)}\sqrt{2}+\frac{5}{9}\mathbf{e}^{(-t)}+\frac{2}{9}\mathbf{e}^{(-t)}\sqrt{2}+\frac{1}{3}t\,\mathbf{e}^{(-t)}+\frac{1}{3}t\,\mathbf{e}^{(-t)}\sqrt{2}$$

```
> y_temp:=soln_temp[2]: y:=unapply(rhs(y_temp),t): y(t);
```

$$\frac{2}{9}\mathbf{e}^{(-4\,t)}-\frac{2}{9}\mathbf{e}^{(-4\,t)}\sqrt{2}+\frac{7}{9}\mathbf{e}^{(-t)}+\frac{2}{9}\mathbf{e}^{(-t)}\sqrt{2}+\frac{2}{3}t\,\mathbf{e}^{(-t)}+\frac{1}{3}t\,\mathbf{e}^{(-t)}\sqrt{2}$$

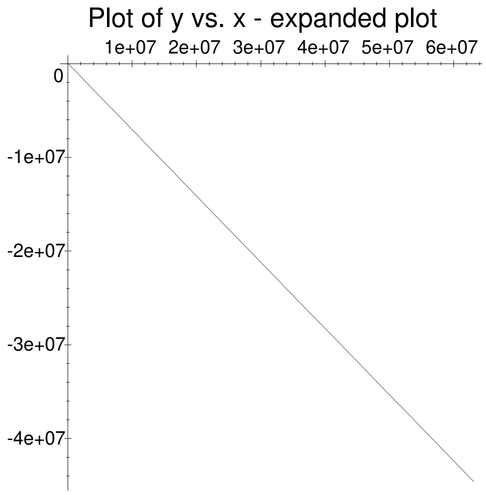
```
> plot([x(t),y(t), t=-1..1], font=[HELVETICA,BOLD,10], title=`Plot of y vs. x`,
```

```
titlefont=[HELVETICA,BOLD,14], axes=NORMAL);
```



Expand the time interval for the above plot, and use the graphical evidence to conjecture what happens as *t* approaches plus and minus ∞
.

```
> plot([x(t),y(t), t=-5..5], font=[HELVETICA,BOLD,10], title=`Plot of y vs. x - expanded
  plot`, titlefont=[HELVETICA,BOLD,14], axes=NORMAL);
```



It appears that both components of the solution will approach infinity as *t* approaches minus ∞, and they will approach zero as *t* approaches plus ∞

```
>
```