COMPUTER PROJECT #5

Math 316
Autumn 1997

Let (x_0, y_0, z_0) be a critical point for the three-dimensional system

$$\begin{aligned} x' &= f(x,y,z) \\ y' &= g(x,y,z) \\ z' &= h(x,y,z). \end{aligned}$$

Similiar to the analysis for two-dimensional systems, the stability of the critical point can be determined by finding the eigenvalues of the matrix

$$A = \begin{bmatrix} f_x & f_y & f_z \\ g_x & g_y & g_z \\ h_x & h_y & h_z \end{bmatrix},$$

where all the derivatives are evaluated at the critical point. This information is necessary in order to do Part 1. below.

Consider the Rössler system

$$\begin{aligned} x' &= -(y+z) \\ y' &= x + 0.36\,y \\ z' &= 0.4\,x - 4.5\,z + xz. \end{aligned}$$

This ODE has solutions which are **chaotic**.

1. Find all of the critical points, and discuss the type and stability of each one.

2. For $a = 0.1$, plot on the same graph the first coordinate $x(t)$ of the solutions corresponding to the initial conditions (2,2,2) and $(2+a, 2, 2)$ from $t = 0$ to $t = 120$. These plots should be in the tx-plane. Observe the time at which the solutions start to differ noticeably.

3. Repeat Part 2. for $a = 0.01$, 0.001, 0.0001. Be consistent in the criterion that you are using to define "noticeable". What conclusions, if any, can you draw?

4. In the xy-plane, plot the trajectory of the solution with the initial condition (2,2,2) from $t = 0$ to $t = 120$. The trajectory appears to self-intersect. Does the trajectory actually intersect itself? Why or why not?

Hand in a print out of your complete computer algebra solutions and your graphs. Cut all sheets down to $8\frac{1}{2}$ by 11 inches and staple in the upper left-hand corner.