Fourth Computing Project
MA375, Fall 1993
Professor Coutsias

This project is due November 13. Plan your work so that if your computer is down a short period, your project will not be late.

This project will count as part of your grade so you must work independently. It is permissible to discuss the project with your instructor, fellow students, and friends. However, the computer programs and report must be done only by the student doing the project.

The project is to write programs to implement

1. Newton's and the secant method for solving a single equation, in order to compare their rates of convergence.

2. Newton's method for a zero of order p (different from 1). You will need to determine p and modify the algorithm to restore quadratic convergence (extra credit).

You will also need to implement the textbook algorithm "zero" in order to have a means of independently solving the problems in (1)-(2), so that you can be assured that your own programs perform correctly. However, near a multiple zero, the routine "zero" will have difficulties (why?).

Test your programs on the following polynomials:

$$P_1(x) = 128x^8 - 256x^6 + 160x^4 - 32x^2 + 1 \quad (1)$$

$$P_2(x) = 256x^9 - 576x^7 + 432x^5 - 120x^3 + 9x + 1 \quad (2)$$

Specifically, you must find as many roots of P_1 and P_2 as you can in the interval $-1 \le x \le 1$. You can use symmetry arguments to cut down on the work. The textbook routine "zero" is adequate for P_1 but P_2 has multiple zeros and it never becomes negative. For it, you must use your own routine. Your routine should include:

(i) User supplied evaluation for function (and derivative, if needed).

(ii) User supplied initial guess.

(iii) User supplied error tolerance and upper bound on the number of iterations.

In all cases display your initial guess, the number of iterations, as well as the final solution to which you converge. After you have the converged result, x^*, go back and compute $\|e^k\| = \|x^k - x^*\|$. Study the behavior of $\frac{\|e^{k+1}\|}{\|e^k\|^\alpha}$ where $\alpha = 2$ for Newton and $\alpha = 1.618...$ for the secant method. Do you observe the expected convergence? If not, try to explain why not.

EXTRA CREDIT: In case you find that the convergence for Newton's method is not quadratic, use the method discussed in class to find what order zero you are trying to compute, and modify Newton's method to restore quadratic convergence.

Your programs should be well written and well documented. Do NOT turn in any listings of library programs, but do turn in your own subroutines and drivers. A well-written description of all your results should be included.