Hw. 3

Supplement

2.2 **5.** Write the complete solutions $x = x_p + x_n$ to these systems, as in equation (4):

$$\begin{bmatrix}1&2&2\\2&4&5\end{bmatrix}\begin{bmatrix}u\\v\\w\end{bmatrix}=\begin{bmatrix}1\\4\end{bmatrix} \qquad \begin{bmatrix}1&2&2\\2&4&4\end{bmatrix}\begin{bmatrix}u\\v\\w\end{bmatrix}=\begin{bmatrix}1\\4\end{bmatrix}.$$

(a) (b)

$$-2\left(\begin{array}{ccc|c}1&2&2&1\\2&4&5&4\end{array}\right) \longrightarrow \left(\begin{array}{ccc|c}1&\overset{*\text{free}}{2}&2&1\\0&0&1&2\end{array}\right)^{-2} \longrightarrow \left(\begin{array}{ccc|c}1&2&0&-3\\0&0&1&2\end{array}\right) \Rightarrow \begin{pmatrix}u\\w\end{pmatrix} = \begin{pmatrix}-3\\2\end{pmatrix} - v\begin{pmatrix}2\\0\end{pmatrix}$$

i.e. $x_p = \begin{pmatrix}-3\\0\\2\end{pmatrix}$, $x_n = \begin{pmatrix}-2\\1\\0\end{pmatrix}$ (set $v=0$ for x_p; ignore rhs. & set $v=1$ for x_n)

(b) $-2\left(\begin{array}{ccc|c}1&2&2&1\\2&4&4&4\end{array}\right) \rightarrow \left(\begin{array}{ccc|c}1&2&2&1\\0&0&0&2\end{array}\right)$ inconsistent, no solution x_p

2.2.**7.** Find the value of c that makes it possible to solve $Ax = b$, and solve it:

$$\begin{aligned} u + v + 2w &= 2\\ 2u + 3v - w &= 5\\ 3u + 4v + w &= c. \end{aligned}$$

$$\begin{array}{c}\\-2\\-3\end{array}\left(\begin{array}{ccc|c}1&1&2&2\\2&3&-1&5\\3&4&1&c\end{array}\right) \longrightarrow \begin{array}{c}\\ \\-1\end{array}\left(\begin{array}{ccc|c}1&1&2&2\\0&1&-5&1\\0&1&-5&c-6\end{array}\right) \longrightarrow \left(\begin{array}{ccc|c}1&1&2&2\\0&1&-5&1\\0&0&0&c-7\end{array}\right) \Rightarrow c=7 \text{ for consistency}$$

$$\longrightarrow \left(\begin{array}{ccc|c}1&1&2&2\\0&1&-5&1\end{array}\right)^{-1} \longrightarrow \left(\begin{array}{ccc|c}1&0&-3&1\\0&1&-5&1\end{array}\right) \Rightarrow \begin{pmatrix}u\\v\end{pmatrix} = \begin{pmatrix}1\\1\end{pmatrix} + w\begin{pmatrix}3\\5\end{pmatrix};\ w \text{ free.}$$

2.2.**8.** Under what conditions on b_1 and b_2 (if any) does $Ax = b$ have a solution?

$$A = \begin{bmatrix}1&2&0&3\\2&4&0&7\end{bmatrix}, \qquad b = \begin{bmatrix}b_1\\b_2\end{bmatrix}.$$

Find two vectors in the nullspace of A, and the complete solution to $Ax = b$.

$$-2\left(\begin{array}{cccc|c}1&2&0&3&b_1\\2&4&0&7&b_2\end{array}\right) \longrightarrow \left(\begin{array}{cccc|c}1&2&0&3&b_1\\0&0&0&1&b_2-2b_1\end{array}\right)^{-3} \longrightarrow$$

consistent

$$\rightarrow \left(\begin{array}{cccc|c}1&\overset{*}{2}&\overset{*}{0}&0&7b_1-3b_2\\0&0&0&1&b_2-2b_1\end{array}\right) \Rightarrow \begin{pmatrix}x_1\\x_4\end{pmatrix} = \begin{pmatrix}7b_1-3b_2\\b_2-2b_1\end{pmatrix} - x_2\begin{pmatrix}2\\0\end{pmatrix} + x_3\begin{pmatrix}0\\0\end{pmatrix}$$

$$\vec{x} = \cancel{\begin{pmatrix}7b_1-3b_2\\b_2-2b\end{pmatrix}}\begin{pmatrix}7b_1-3b_2\\0\\0\\b_2-2b_1\end{pmatrix} - \alpha\underbrace{\begin{pmatrix}2\\-1\\0\\0\end{pmatrix} + \beta\begin{pmatrix}0\\0\\1\\0\end{pmatrix}}_{\text{null vectors}}$$

α, β arbitrary

2.3. **2.** Find the largest possible number of independent vectors among

$$v_1 = \begin{bmatrix}1\\-1\\0\\0\end{bmatrix} \quad v_2 = \begin{bmatrix}1\\0\\-1\\0\end{bmatrix} \quad v_3 = \begin{bmatrix}1\\0\\0\\-1\end{bmatrix} \quad v_4 = \begin{bmatrix}0\\1\\-1\\0\end{bmatrix} \quad v_5 = \begin{bmatrix}0\\1\\0\\-1\end{bmatrix} \quad v_6 = \begin{bmatrix}0\\0\\1\\-1\end{bmatrix}.$$

This number is the ______ of the space spanned by the v's.

The number is $\underline{4}$: $\dim \operatorname{span}\{v_i\}_{i=1}^{6} \le 4$

in fact the dimension of the span $= 3$

$$\begin{pmatrix}1&1&1&0&0&0\\-1&0&0&1&1&0\\0&-1&0&-1&0&1\\0&0&-1&0&-1&-1\end{pmatrix} \rightarrow \begin{pmatrix}1&1&1&0&0&0\\0&1&1&1&1&0\\0&-1&0&-1&0&1\\0&0&-1&0&-1&-1\end{pmatrix} \rightarrow \begin{pmatrix}1&1&1&0&0&0\\0&1&1&1&1&0\\0&0&1&0&1&1\\0&0&-1&0&-1&-1\end{pmatrix} \rightarrow \begin{pmatrix}1&1&1&0&0&0\\0&1&1&1&1&0\\0&0&1&0&1&1\\0&0&0&0&0&0\end{pmatrix} \Big\}\ \{v_1, v_2, v_3\} \text{ independent}$$