2.3. **6.** Choose three independent columns of U. Then make two other choices. Do the same for A. You have found bases for which spaces?

$$U = \begin{bmatrix} 2 & 3 & 4 & 1 \\ 0 & 6 & 7 & 0 \\ 0 & 0 & 0 & 9 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{and} \quad A = \begin{bmatrix} 2 & 3 & 4 & 1 \\ 0 & 6 & 7 & 0 \\ 0 & 0 & 0 & 9 \\ 4 & 6 & 8 & 2 \end{bmatrix}.$$

$$U = \begin{bmatrix} 2 & 3 & 4 & 1 \\ 0 & 6 & 7 & 0 \\ 0 & 0 & 0 & 9 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

pivotal columns ⇒ independent (1, 2, 4)

$$A = \begin{bmatrix} 2 & 3 & 4 & 1 \\ 0 & 6 & 7 & 0 \\ 0 & 0 & 0 & 9 \\ 4 & 6 & 8 & 2 \end{bmatrix} \begin{matrix} \\ \\ \\ -2 \end{matrix} \longrightarrow \begin{bmatrix} 2 & 3 & 4 & 1 \\ 0 & 6 & 7 & 0 \\ 0 & 0 & 0 & 9 \\ 0 & 0 & 0 & 0 \end{bmatrix} = U \Rightarrow$$ columns (1, 2, 4) of A are independent

(either U or A also have columns (1, 3, 4) indep.)

These form a basis for $R(U)$ / $R(A)$ resp.

2.3. **10.** Find two independent vectors on the plane $x+2y-3z-t=0$ in $\mathbf{R}^4$. Then find three independent vectors. Why not four? This plane is the nullspace of what matrix?

$$\begin{bmatrix} 1 & 2 & -3 & -1 \end{bmatrix} \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} = 0 \Rightarrow x = -2y + 3z + t :$$

$$\begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} = \alpha \begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 3 \\ 0 \\ 1 \\ 0 \end{pmatrix} + \gamma \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$: 3 null vectors (matrix has rank 1: 1 nonzero row, 4 columns). 3 free variables

Now: $$\begin{pmatrix} 1 & 2 & -3 & -1 \end{pmatrix} \begin{pmatrix} -2 & 3 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \end{pmatrix} = \vec{0}$$

$$\Rightarrow \underbrace{\begin{pmatrix} -2 & 1 & 0 & 0 \\ 3 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}}\begin{pmatrix} 1 \\ 2 \\ -3 \\ -1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix};$$

$\begin{pmatrix} 1 \\ 2 \\ -3 \\ -1 \end{pmatrix}$ is null vector of