18.06 **Problem Set 1**

due: Wednesday, 14 February 2001

1. (10pts)

   (a) Write down the most general equation of a plane in three dimensions.

   (b) Find the normal vector to the plane of part 1a.

   (c) Find the equation of the plane through $P_0 : (1, 0, 1)$ with normal vector $\mathbf{n} = (2, -1, 3)$.

   (d) Find the equation of the plane through the three points: $P : (1, 1, 2)$, $Q : (2, 3, 4)$, $R : (2, 1, 1)$.

   (e) Find the distance from the plane in part 1d to the origin (i.e. the shortest distance from a point in the plane to the origin).

2. (10pts) When you try to find the intersection of three planes in $\mathbb{R}^3$, what are the possible geometric objects that result? Try to draw an example of each case.

3. (10pts)

   (a) A 3×3 upper triangular matrix is of the form

   $$\begin{pmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{pmatrix}$$

   Show that the product of two upper triangular matrices is an upper triangular matrix.

   (b) Calculate the product of two upper triangular matrices with vanishing diagonal entries ($a = d = f = 0$).

   (c) Find two 2×2 matrices γ^+ and γ^- which satisfy

   $$\begin{aligned} (\gamma^+)^2 = (\gamma^-)^2 &= 0 \\ \gamma^+\gamma^- + \gamma^-\gamma^+ &= \mathbf{I} \end{aligned}$$

   where $\mathbf{I}$ is the 2×2 identity matrix.

4. (10pts) Find all solutions of the following systems of linear equations:

   (a)

   $$\begin{aligned} x_1 - 2x_2 + x_3 - x_4 &= 8 \\ 3x_1 - 6x_2 + 2x_3 &= 18 \\ x_3 - 2x_4 &= 5 \end{aligned}$$

   (b)

   $$\begin{aligned} x_1 - 3x_2 + x_3 &= 2 \\ 3x_1 - 8x_2 + 2x_3 &= 5 \\ 3x_1 - 7x_2 + x_3 &= 1 \end{aligned}$$

(c)

$$\begin{aligned} x_1 - 2x_3 + x_4 &= 6 \\ 2x_1 - x_2 + x_3 - 3x_4 &= 0 \\ 9x_1 - 3x_2 - x_3 - 7x_4 &= 4 \end{aligned}$$

Give a solution of this system with $x_2 = -3$.

(d)

$$\begin{aligned} x_2 + 2x_3 &= 0 \\ x_1 + 3x_2 + x_3 &= 0 \\ x_1 + x_2 - 3x_3 &= 0 \end{aligned}$$

5. (10pts) Consider the system

$$\begin{aligned} x_1 + x_2 + x_3 &= -1 \\ 2x_1 + x_2 + ax_3 &= 1 \\ 3x_1 + x_2 + x_3 &= b \end{aligned}$$

where a and b are real numbers. For which values of a and b does the system have

(a) no solutions;

(b) exactly one solution;

(c) finitely many, but at least two solutions;

(d) infinitely many solutions?