

18.06 Problem Set 4

due: Wednesday, 7 March 2001

1. (10pts.)

(a) In each of the following cases determine whether or not the given vectors span \mathbb{R}^3 and are linearly independent.

i. $(1, 3, -1), (2, 1, 1);$

ii. $(1, 3, -1), (2, 1, 1), (-1, 7, 5);$

iii. $(1, 3, -1), (2, 1, 1), (2, -5, 4), (1, 2, 5)$

(b) Find a (linear) equation in a, b, c that holds if and only if $(a, b, c) \in \text{Sp}((1, 3, -1), (2, 1, 1))$.

(c) (**optional**) Let V be the VS \mathbb{C}^3 over \mathbb{C} . Do the vectors $(i, 3 + i, i), (1 - i, 2, 2 + i), (-2 - 2i, 4i, 1 - i)$ span V ? Are they linearly independent?

2. (10pts.) Let U and W be subspaces of \mathbb{R}^4 given by the following conditions

$$U = \{(a, b, c, d) | b + c + 2d = 0, b = c\}, \quad W = \{(a, b, c, d) | a + b = 0, c = 2d\}$$

Find bases for each of U, W . Determine the dimension of $U \cap W$ and $U + W$.

3. (10pts.) Find the row and column ranks of the matrices

$$\begin{pmatrix} 1 & 3 & 1 & -2 & -3 \\ 1 & 4 & 3 & -1 & -4 \\ 2 & 3 & -4 & -7 & -3 \\ 3 & 8 & 1 & -7 & -8 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 1 & 3 \\ 0 & -2 \\ 5 & -1 \\ -2 & 3 \end{pmatrix}$$

4. (10pts.)

(a) Find an equation in a and b that is satisfied if and only if the matrix

$$\begin{pmatrix} 3 & 2 & 5 \\ 1 & a & -1 \\ -1 & 3 & b \end{pmatrix}$$

does not have row rank 3.

(b) Find an equation in b, c, d that is satisfied if and only if the matrices A, B below have the same row rank.

$$A = \begin{pmatrix} 1 & 2 & -3 \\ 1 & 1 & 0 \\ -2 & -1 & 3 \\ -1 & 4 & -2 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 2 & -3 & 0 \\ 1 & 1 & 0 & b \\ -2 & -1 & 3 & c \\ -1 & 4 & -2 & d \end{pmatrix}$$

5. (10pts.) From Strang's book, section 3.5, do problems 30 and 34.