18.06 **Hints and Answers to Problem Set 7**

2. (a) 15,11.

   (b) For the first problem, $t = -2, 2$, or -4. For the second problem, $t = 0, b-c$, or $(a+b+c)/2$.

3. Expanding by the first column,

$$\det A_n = 1 \cdot \det A_{n-1} + 1 \cdot \det A_{n-1}.$$

   Now use induction. For $n = 1$, $\det A_1 = 1 = 2^0$. Assume the formula holds for n. Then $\det A_{n+1} = 2 \cdot \det A_n = 2 \cdot 2^{n-1} = 2^n$. Thus the formula holds for $n+1$.

4. (a) 60.

   (b) Use $\cos\gamma = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ to find that the answer is 0.

5. We have

$$\begin{pmatrix} A & 0 \\ 0 & I_n \end{pmatrix} \cdot \begin{pmatrix} I_k & 0 \\ C & D \end{pmatrix} = \begin{pmatrix} A \cdot I_k + 0 \cdot C & A \cdot 0 + 0 \cdot D \\ 0 \cdot I_k + I_n \cdot C & 0 \cdot 0 + I_n \cdot D \end{pmatrix} = \begin{pmatrix} A & 0 \\ C & D \end{pmatrix}.$$

   Now let

$$A_m = \begin{pmatrix} A & 0 \\ 0 & I_m \end{pmatrix}$$

   Expanding along the last row, we find that $\det A_m = \det A_{m-1}$ for $m > 0$. Using this formula repeatedly, we find that $\det A_n = \det A_0 = \det A$.

   Now define C_ℓ for $0 \le \ell \le k$ to be the n by ℓ matrix, which has the last ℓ columns of the matrix C. Further, define

$$D_\ell = \begin{pmatrix} I_\ell & 0 \\ C_\ell & D \end{pmatrix}.$$

   Expanding along the first row, we find that $\det D_\ell = \det D_{\ell-1}$ for $1 \le \ell \le k$. Using this formula repeatedly, we find that $\det D_k = \det D_0 = \det D$. So finally,

$$\det \begin{pmatrix} A & 0 \\ C & D \end{pmatrix} = \det \begin{pmatrix} A & 0 \\ 0 & I_n \end{pmatrix} \cdot \det \begin{pmatrix} I_k & 0 \\ C & D \end{pmatrix} = (\det A) \cdot (\det D).$$