18.06 **Problem Set 2**

due: Thursday, 22 February 2001

1. (10pts.) Let $A = \begin{pmatrix} 1 & 2 & -1 \\ 2 & -3 & 2 \\ -1 & 12 & -7 \end{pmatrix}$ and $\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$.

   Recall from 18.03 that the general solution of a non-homogeneous linear o.d.e. can be written as the sum of the general solution of the homogeneous equation and a particular solution. This same idea carries over to linear matrix equations, as shown in this problem.

   (a) Solve $A\mathbf{x} = \mathbf{0}$.

   (b) Let $\mathbf{b}^T = (1, 1, 1)$. Find all solutions of $A\mathbf{x} = \mathbf{b}$ in the form $\mathbf{p} + \mathbf{h}$, where $\mathbf{p}$ is a particular solution of $A\mathbf{x} = \mathbf{b}$.

   (c) Find a vector $\mathbf{c}$ such that $A\mathbf{x} = \mathbf{c}$ has no solutions.

2. (10pts.) Show that if A is $m \times n$ and B is $n \times p$, then $B^T A^T$ is defined and $B^T A^T = (AB)^T$. **Note:** Recall that if the ij-entry of matrix A is a_{ij}, then the ij-entry of matrix A^T is a_{ji}. Furthermore, if the ij-entry of the matrix B is b_{ij}, then the ij-entry of the product AB is $\sum_{k=1}^{n} a_{ik} b_{kj}$.

3. (10pts.) Find 2×2 matrices A, B, C such that for any $2 \times n$ matrix D,

   (a) AD is D with its first row multiplied by 2;

   (b) BD is D with its rows interchanged;

   (c) CD is obtained from D by adding the first row to the second.

4. (10pts.) In this question, all matrices are $n \times n$. If $AB = BA = I$ then B is called the *inverse* of A and denoted by A^{-1}. Prove the following facts:

   (a) If $CA = I$ and $AB = I$, then $B = C = A^{-1}$.

   (b) $(A^{-1})^{-1} = A$.

   (c) If A and B are invertible, then so is AB and $(AB)^{-1} = B^{-1}A^{-1}$.

   (d) If A is invertible then for any n-vector $\mathbf{b}$, the equations $A\mathbf{x} = \mathbf{b}$ have a unique solution.

   (e) Find the inverse of $A = \begin{pmatrix} 3 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & -1 & 2 \end{pmatrix}$.

5. (10pts.)

   (a) Strang, Section 2.6, problems 2, 3.

   (b) Type in the `slu` program from Strang p. 82-83, and use it to check your answers to 5a.

   (c) Strang, Section 2.6, problem 26. Run your program on the matrices of 5a.