18.06 **Problem Set 3**

due: Tuesday, 27 February 2001

Hint: In order to do these problems, you may find it useful to look at the list of rules on page 107 in Strang's book. Also remember that vector spaces need to be closed under addition and scalar multiplication, i.e. if $\mathbf{x}$ and $\mathbf{y}$ are elements of the space, then $\mathbf{x}+\mathbf{y}$ also needs to be an element of the space. Furthermore, if r is a number, then $r\mathbf{x}$ needs to be an element of the space.

1. (10pts.) Which of the following sets are vector spaces over $\mathbb{R}$? Justify your answers.
   (a) The set $\mathbb{R}^2$ with the usual addition, but scalar multiplication defined by $r(x,y) = (rx, y)$.
   (b) The set $\mathbb{R}^2$ with the usual scalar multiplication, but addition defined by $(x,y)+(u,v) = (y+v, x+u)$.
   (c) The set of all solutions of the differential equation $(\sin x)y'' + (x^2-2)y' + (\cosh x)^2 y = 0$. (**Warning:** Do not try to solve the equation).
   (d) The set of vectors $(a,b,c,d) \in \mathbb{R}^4$ such that $ab \geq 0$ with the usual addition and scalar multiplication.
   (e) The set of all functions from $\mathbb{R}$ to $\mathbb{R}$ with the usual scalar multiplication, but with addition defined by $(f+g)(x) = \max\{f(x), g(x)\}$ for all $x \in \mathbb{R}$.

2. (10pts.) Let A be the matrix $\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$.
   (a) For which vectors $\mathbf{b} \in \mathbb{R}^2$ is the set $\{\mathbf{x} \in \mathbb{R}^3 | A\mathbf{x} = \mathbf{b}\}$ a subspace of $\mathbb{R}^3$?
   (b) Is the set of vectors $\mathbf{b} \in \mathbb{R}^2$ for which the equation $A\mathbf{x} = \mathbf{b}$ has a solution a subspace of $\mathbb{R}^2$?
   (c) Is the set defined in problem 2b a subspace of $\mathbb{R}^3$?

   Justify your answers.

3. (10pts.) Strang, section 3.2, problems 23,24. Section 3.4, problem 3.