

## 18.06 Problem Set 4

due: Wednesday, 7 March 2001

1. (10pts.)

(a) In each of the following cases determine whether or not the given vectors span  $\mathbb{R}^3$  and are linearly independent.

- (1, 3, -1), (2, 1, 1);
- (1, 3, -1), (2, 1, 1), (-1, 7, 5);
- (1, 3, -1), (2, 1, 1), (2, -5, 4), (1, 2, 5)

(b) Find a (linear) equation in  $a, b, c$  that holds if and only if  $(a, b, c) \in \text{Sp}((1, 3, -1), (2, 1, 1))$ .

(c) (optional) Let  $V$  be the VS  $\mathbb{C}^3$  over  $\mathbb{C}$ . Do the vectors  $(i, 3 + i, i)$ ,  $(1 - i, 2, 2 + i)$ ,  $(-2 - 2i, 4i, 1 - i)$  span  $V$ ? Are they linearly independent?

2. (10pts.) Let  $U$  and  $W$  be subspaces of  $\mathbb{R}^4$  given by the following conditions

$$U = \{(a, b, c, d) | b + c + 2d = 0, b = c\}, \quad W = \{(a, b, c, d) | a + b = 0, c = 2d\}$$

Find bases for each of  $U, W$ . Determine the dimension of  $U \cap W$  and  $U + W$ .

3. (10pts.) Find the row and column ranks of the matrices

$$\begin{pmatrix} 1 & 3 & 1 & -2 & -3 \\ 1 & 4 & 3 & -1 & -4 \\ 2 & 3 & -4 & -7 & -3 \\ 3 & 8 & 1 & -7 & -8 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 1 & 3 \\ 0 & -2 \\ 5 & -1 \\ -2 & 3 \end{pmatrix}$$

4. (10pts.)

(a) Find an equation in  $a$  and  $b$  that is satisfied if and only if the matrix

$$\begin{pmatrix} 3 & 2 & 5 \\ 1 & a & -1 \\ -1 & 3 & b \end{pmatrix}$$

does not have row rank 3.

(b) Find an equation in  $b, c, d$  that is satisfied if and only if the matrices  $A, B$  below have the same row rank.

$$A = \begin{pmatrix} 1 & 2 & -3 \\ 1 & 1 & 0 \\ -2 & -1 & 3 \\ -1 & 4 & -2 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 2 & -3 & 0 \\ 1 & 1 & 0 & b \\ -2 & -1 & 3 & c \\ -1 & 4 & -2 & d \end{pmatrix}$$

5. (10pts.) From Strang's book, section 3.5, do problems 30 and 34.