18.06 **Problem Set 4**

due: Wednesday, 7 March 2001

1. (10pts.)

   (a) In each of the following cases determine whether or not the given vectors span $\mathbb{R}^3$ and are linearly independent.

      i. $(1,3,-1),\ (2,1,1)$;

      ii. $(1,3,-1),\ (2,1,1),\ (-1,7,5)$;

      iii. $(1,3,-1),\ (2,1,1),\ (2,-5,4),\ (1,2,5)$

   (b) Find a (linear) equation in a, b, c that holds if and only if $(a,b,c) \in \mathrm{Sp}((1,3,-1),(2,1,1))$.

   (c) (**optional**) Let V be the VS $\mathbb{C}^3$ over $\mathbb{C}$. Do the vectors $(i, 3+i, i)$, $(1-i, 2, 2+i)$, $(-2-2i, 4i, 1-i)$ span V? Are they linearly independent?

2. (10pts.) Let U and W be subspaces of $\mathbb{R}^4$ given by the following conditions

$$U = \{(a,b,c,d) | b+c+2d = 0,\ b = c\}, \qquad W = \{(a,b,c,d) | a+b = 0,\ c = 2d\}$$

   Find bases for each of U, W. Determine the dimension of $U \cap W$ and $U + W$.

3. (10pts.) Find the row and column ranks of the matrices

$$\begin{pmatrix} 1 & 3 & 1 & -2 & -3 \\ 1 & 4 & 3 & -1 & -4 \\ 2 & 3 & -4 & -7 & -3 \\ 3 & 8 & 1 & -7 & -8 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 1 & 3 \\ 0 & -2 \\ 5 & -1 \\ -2 & 3 \end{pmatrix}$$

4. (10pts.)

   (a) Find an equation in a and b that is satisfied if and only if the matrix

$$\begin{pmatrix} 3 & 2 & 5 \\ 1 & a & -1 \\ -1 & 3 & b \end{pmatrix}$$

   does not have row rank 3.

   (b) Find an equation in b, c, d that is satisfied if and only if the matrices A, B below have the same row rank.

$$A = \begin{pmatrix} 1 & 2 & -3 \\ 1 & 1 & 0 \\ -2 & -1 & 3 \\ -1 & 4 & -2 \end{pmatrix}, \qquad B = \begin{pmatrix} 1 & 2 & -3 & 0 \\ 1 & 1 & 0 & b \\ -2 & -1 & 3 & c \\ -1 & 4 & -2 & d \end{pmatrix}$$

5. (10pts.) From Strang's book, section 3.5, do problems 30 and 34.