18.06 **Problem Set 5**

due: Wednesday, 14 March 2001

1. (10pts.) Let A be an $m \times n$ matrix and B be an $n \times p$ matrix.

   (a) Show that for a *row vector* $\mathbf{v}$ in $\mathbb{R}^n$, $\mathbf{v}B$ is a linear combination of the rows of B, and $A\mathbf{v}^T$ is a linear combination of the columns of A.

   (b) Show that the row space of AB is contained in the row space of B.

   (c) Prove that $\text{Rank}(AB) \leq \text{Rank}(B)$ and also $\text{Rank}(AB) \leq \text{Rank}(A)$.

   (d) For $m \geq n$, show that $A^T A$ is nonsingular if and only if A has full rank.

2. (10pts.) From Strang's book, section 4.1, do problems 9 and 22.

3. (10pts.) From Strang's book, section 4.2, do problems 2,4 and 13

4. (10pts.) From Strang's book, section 4.2, do problems 23 and 28.

5. (10pts.) In this problem you should use Matlab to do the calculations. Find the closest vector to
   `b=[1 2 3 4 5 6]'`
   in the nullspace of
   `A=[-1 1 0 0 0 0;-1 0 1 0 0 0;-1 0 0 1 0 0;-1 0 0 0 1 0;-1 0 0 0 0 1]`
   Guess before you start, then:

   (a) Use the matlab command `a = null(A,'r')` to work out the nullspace of A. Describe the nullspace. What happens if you ask for `null(A)`?

   (b) Compute the projection matrix P.

   (c) Work out `p = P * b`.

   (d) Confirm that `e = p-b` is orthogonal to the space you're projecting onto.