18.06 **Problem Set 7**

due: Wednesday, 11 April 2001

1. (10pts.) From Strang, section 4.4, do problems 22 and 23.

2. (10pts.)

   (a) Evaluate the determinants $\begin{vmatrix} 0 & -1 & 2 & 1 \\ -4 & 3 & -3 & 5 \\ 1 & 0 & 0 & 1 \\ -1 & 1 & 0 & 1 \end{vmatrix}$ and $\begin{vmatrix} 2 & 1 & 0 & 1 \\ -1 & 2 & 1 & 0 \\ 1 & 2 & 5 & 3 \\ 0 & -1 & 1 & 1 \end{vmatrix}$.

   (b) Solve for t the equations $\begin{vmatrix} t+3 & -1 & 1 \\ 5 & t-3 & 1 \\ 6 & -6 & t+4 \end{vmatrix} = 0$ and $\begin{vmatrix} a-t & b-t & c \\ a-t & c & b-t \\ a & b-t & c-t \end{vmatrix} = 0$.

3. (10pts.) Let A_n be the $n \times n$-matrix

$$\begin{pmatrix} 1 & 1 & 1 & \cdots & 1 & 1 & 1 \\ -1 & 1 & 1 & \cdots & 1 & 1 & 1 \\ 0 & -1 & 1 & \cdots & 1 & 1 & 1 \\ & & \ddots & \ddots & & & \vdots \\ & & & \ddots & \ddots & & \vdots \\ 0 & 0 & 0 & \cdots & -1 & 1 & 1 \\ 0 & 0 & 0 & \cdots & 0 & -1 & 1 \end{pmatrix}$$

   Show that $\det A_n = 2^{n-1}$.

4. (10pts.)

   (a) Evaluate the determinant

$$\begin{vmatrix} 1 & 1 & 3 & 0 & 2 \\ 3 & 1 & 0 & 1 & 2 \\ 0 & 1 & 3 & 0 & 2 \\ 4 & -2 & 3 & 1 & 0 \\ 5 & 1 & 0 & 0 & 6 \end{vmatrix}.$$

   (b) Suppose that $\alpha + \beta = \gamma$. Evaluate

$$\begin{vmatrix} 1 & \cos\alpha & \cos\beta \\ \cos\alpha & 1 & \cos\gamma \\ \cos\beta & \cos\gamma & 1 \end{vmatrix}.$$

5. (10pts.) Prove the following:

   Theorem *Let A be a k by k matrix, let D have size n by n and let C have size n by k. Then*

$$\det \begin{pmatrix} A & 0 \\ C & D \end{pmatrix} = (\det A) \cdot (\det D).$$

Hint: First show that

$$\begin{pmatrix} A & 0 \\ 0 & I_n \end{pmatrix} \cdot \begin{pmatrix} I_k & 0 \\ C & D \end{pmatrix} = \begin{pmatrix} A & 0 \\ C & D \end{pmatrix}.$$