18.06 **Problem Set 8**

due: Wednesday, 18 April 2001

1. (10pts.) Let
$$A = \begin{pmatrix} 3/5 & 4/5 \\ 4/5 & -3/5 \end{pmatrix}.$$
Find the trace, determinant and inverse of A. Also find its eigenvalues and corresponding eigenvectors.

2. (10pts.) Let
$$A = \begin{pmatrix} -3 & 2 & 4 \\ 2 & -6 & 2 \\ 4 & 2 & -3 \end{pmatrix}.$$
   (a) Find the trace and determinant of A.

   (b) Using the cofactor formula from page 232 of Strang's book, compute the inverse of A (you will not receive credit if you find A^{-1} by any other method).

   (c) Given that one of the eigenvalues of A is $\lambda_1 = 2$, find the remaining two eigenvalues and an eigenvector for each eigenvalue.

3. (10pts.) Find an *orthogonal* (4×4) matrix P, and a diagonal matrix D, such that $P^T AP = D$, where A is the matrix
$$\begin{pmatrix} 1 & 3 & 3 & -3 \\ 3 & 1 & -3 & 3 \\ 3 & -3 & 1 & 3 \\ -3 & 3 & 3 & 1 \end{pmatrix}.$$

4. (10pts.) For each of the following statements, prove that it is true or give an example to show it is false. Throughout, A is an $n \times n$ matrix with real entries, unless otherwise indicated, and "ew" stands for eigenvalue. (This comes from the German "Eigenwert". The corresponding abbreviation for eigenvector is "ev", from "Eigenvektor.")

   (a) If λ is an ew of A and $\mu \in \mathbb{C}$, then $\lambda - \mu$ is an ew of $A - \mu I$.

   (b) If λ is an ew of A, then so is $-\lambda$.

   (c) If λ is an ew of A and A is nonsingular, then λ^{-1} is an ew of A^{-1}.

   (d) If all ew's of A are zero, then $A = 0$.

   (e) If A is diagonalisable and all its ew's are equal, then A is diagonal.

5. From Strang's book, do problems 5. and 6. of section 6.3.