18.06 **Hints and Answers to Problem Set 3**

1. (a) Not a VS. It does not satisfy the distributive axioms: $(1+1)(1,1) = 2(1,1) = (2,1) \neq (2,2) = 1(1,1) + 1(1,1)$.

   (b) Not a VS. It does not satisfy the rules for associativity: $((1,0) + (0,2)) + (1,2) = (2,1) + (1,2) = (3,3) \neq (1,5) = (1,0) + ((0,2) + (1,2))$.

   (c) This is a VS: If $y = 0$ then $y' = y'' = 0$ and $(\sin x) \cdot 0 + (x^2 - 2) \cdot 0 + (\cosh x)^2 \cdot 0 = 0$ so 0 is a solution. Suppose y_1 and y_2 are solutions. Then $(\sin x)(y_1 + y_2)'' + (x^2 - 2)(y_1 + y_2)' + (\cosh x)^2(y_1 + y_2) = \big((\sin x)y_1'' + (x^2 - 2)y_1' + (\cosh x)^2 y_1\big) + \big((\sin x)y_2'' + (x^2 - 2)y_2' + (\cosh x)^2 y_2\big) = 0 + 0 = 0$. So $y_1 + y_2$ is a solution. The other axioms also check.

   (d) Not a VS. It is not closed under addition: $(1,2,0,0)$ and $(-2,-1,0,0)$ are both in the set, but their sum $(-1,1,0,0)$ is not.

   (e) Not a VS. There does not exist a function with the zero property: For suppose f were a zero, then choose g so that $g(x) = f(x)$ for $x \neq 0$, but $g(0) = f(0) - 1$. Then $\max\{f, g\} = f \neq g$, which contradicts the zero property of f.

2. (a) This is a subspace iff $\mathbf{b} = \mathbf{0}$. In that case, $A\mathbf{0} = \mathbf{0}$; $A\mathbf{x} = \mathbf{0}$ and $A\mathbf{y} = \mathbf{0}$ imply that $A(r\mathbf{x}) = r(A\mathbf{x}) = r\mathbf{0} = \mathbf{0}$ and $A(\mathbf{x} + \mathbf{y}) = A\mathbf{x} + A\mathbf{y} = \mathbf{0} + \mathbf{0} = \mathbf{0}$. If $\mathbf{b} \neq \mathbf{0}$, then $A\mathbf{0} = \mathbf{0} \neq \mathbf{b}$. (The other conditions also fail.)

   (b) This is also a subspace, because $A\mathbf{0} = \mathbf{0}$, so it contains $\mathbf{0}$. If $\mathbf{b}, \mathbf{c}$ are in the set then choose $\mathbf{x}, \mathbf{y}$ so that $A\mathbf{x} = \mathbf{b}$, $A\mathbf{y} = \mathbf{c}$. Then $A(r\mathbf{x}) = r\mathbf{b}$ and $A(\mathbf{x} + \mathbf{y}) = \mathbf{b} + \mathbf{c}$.

   (c) This set is not a subset of $\mathbb{R}^3$. So it cannot be a subspace.