18.06 **Hints and Answers to Problem Set 4**

1. (a) i. They are linearly independent, but do not span $\mathbb{R}^3$. (there's only two of them.)
   ii. They are linearly independent and span $\mathbb{R}^3$.
   iii. They are not linearly independent (there's four of them and they are vectors in $\mathbb{R}^3$: cf. the Too Many Vectors Theorem), but they do span $\mathbb{R}^3$.

   (b) $4a - 3b - 5c = 0$.

   (c) They do not span $\mathbb{C}^3$. ($\mathrm{Sp}(v_1, v_2, v_3) = \{(4z, 8w, (1+4i)z + (3+i)w)\}$. They are not linearly independent, either: $(2-2i)v_1 - 2v_2 + (1+i)v_3 = 0$.

2. A basis of U is $\{(1,0,0,0), (0,-1,-1,1)\}$; a basis of W is $\{(1,-1,0,0), (0,0,2,1)\}$. So both have dimension 2.

   A vector in $U \cap W$ must satisfy $b+c+d = 0$, $b-c = 0$, $c-2d = 0$. The only solution is $\mathbf{0}$, so the dimension is 0. This is equivalent to saying that the nullspace of the matrix, whose columns are the basis vectors,

$$\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & -1 & 0 & 2 \\ 0 & 1 & 0 & 1 \end{pmatrix}$$

   consists only of $\mathbf{0}$. By the fundamental theorem of linear algebra, part 1, $U + W = \mathrm{Span}\{U, W\} = \mathbb{R}^4$ of dimension 4.

3. All four ranks are 2.

4. (a) $3ab + 5a - 2b + 26 = 0$.

   (b) $3b + 13c - 6d = 0$.